

Cooperation of customers in traveling salesman problems with profits

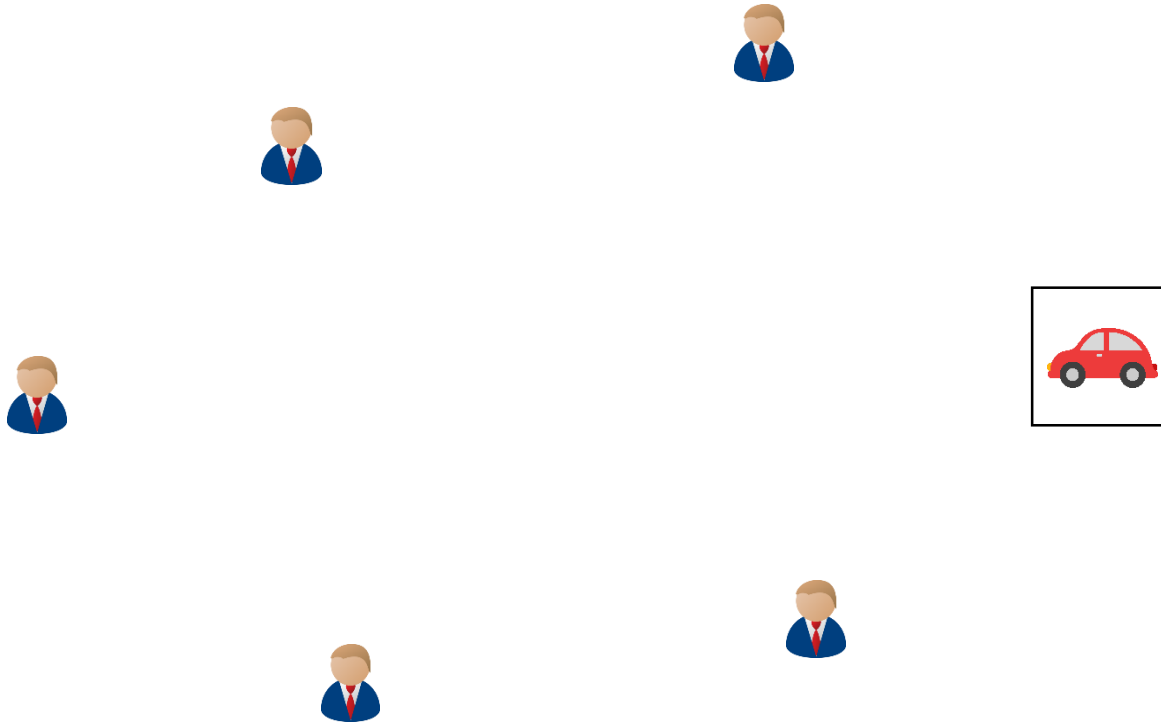
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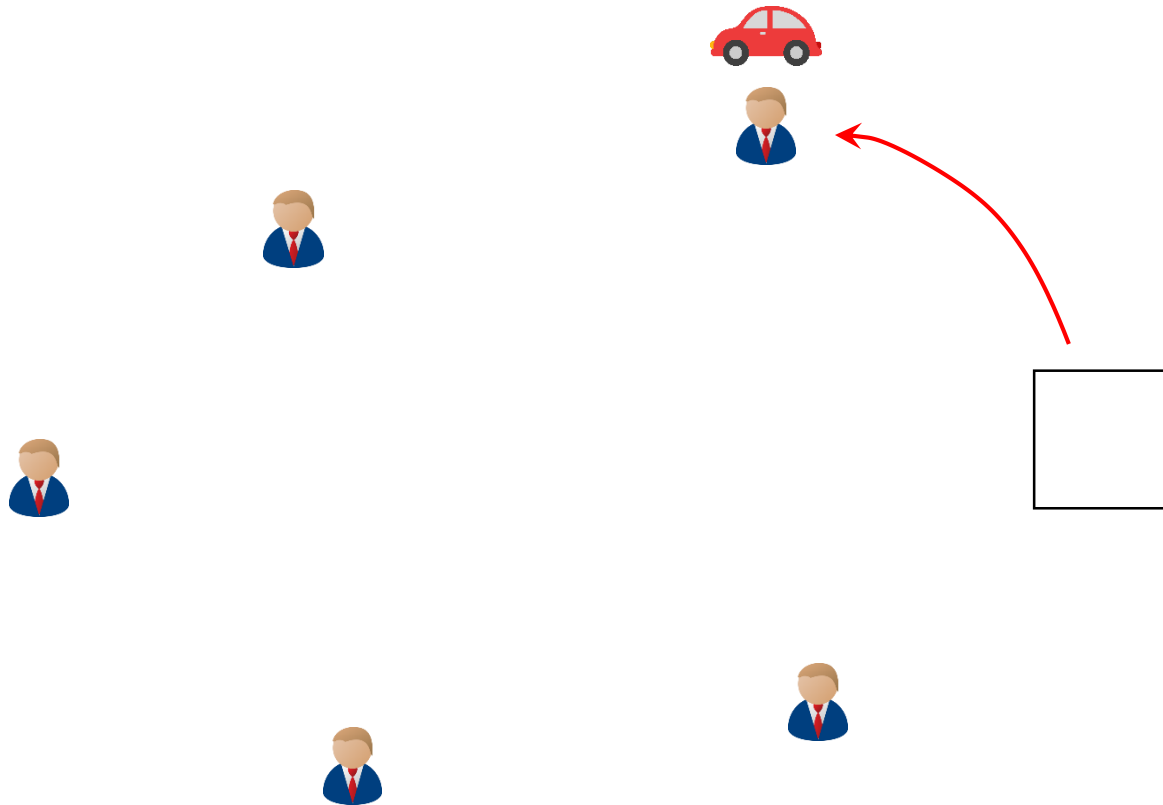
Kurt Jörnsten NHH Norwegian School of Economics
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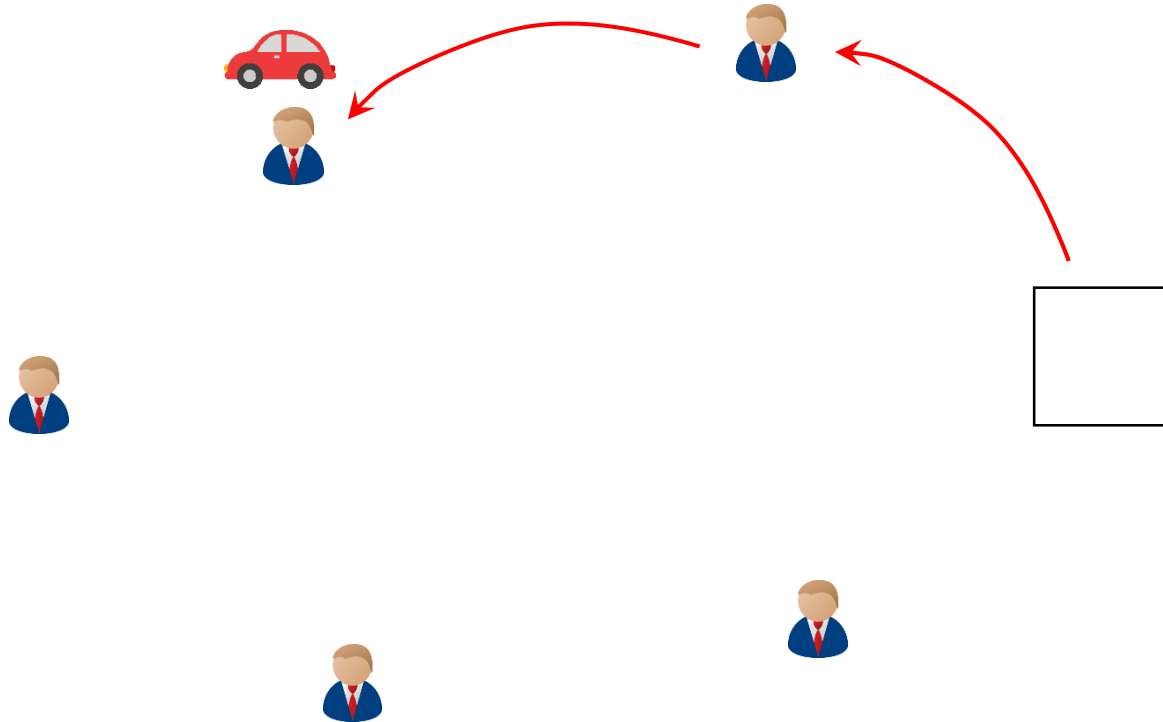
traveling salesman problem



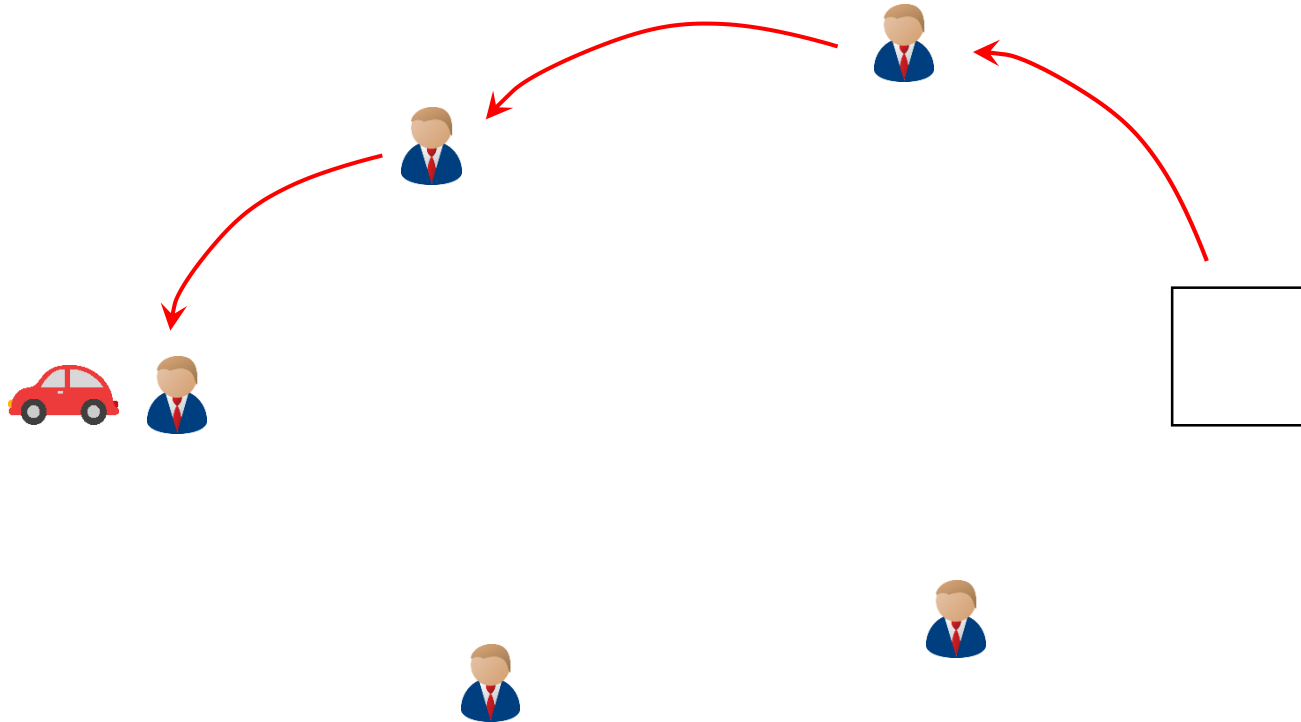
traveling salesman problem



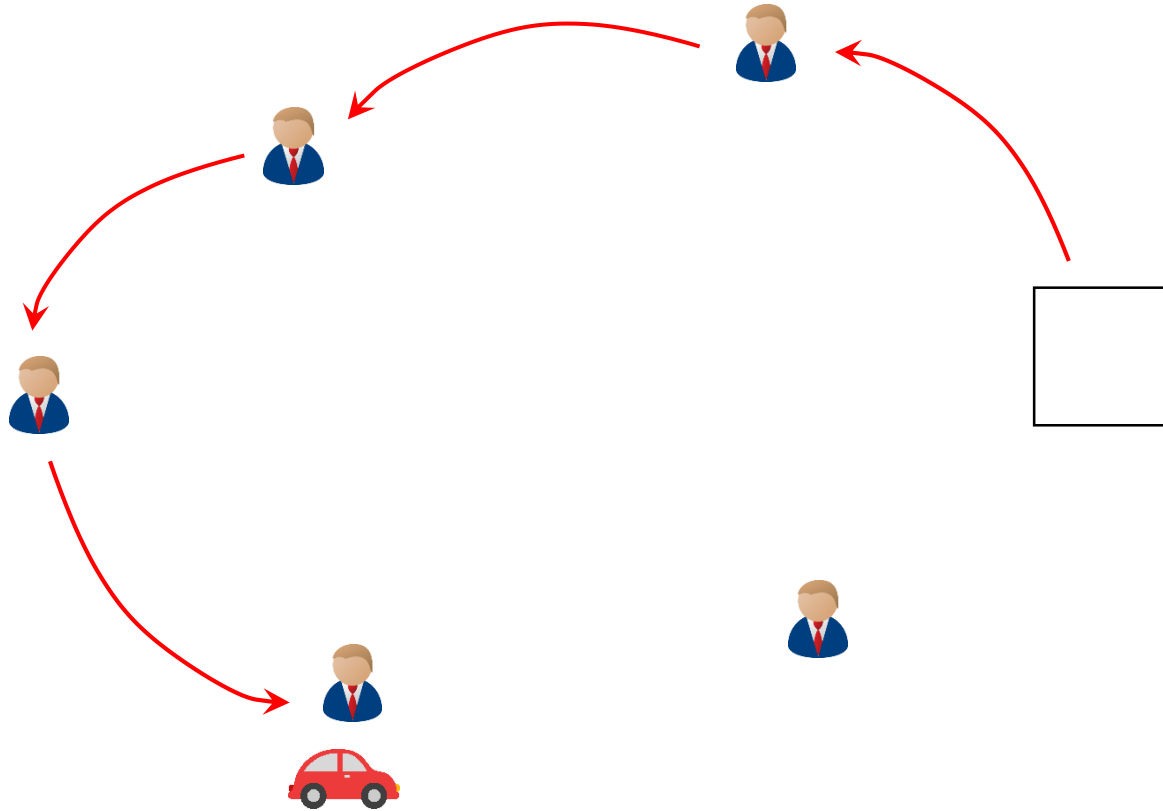
traveling salesman problem



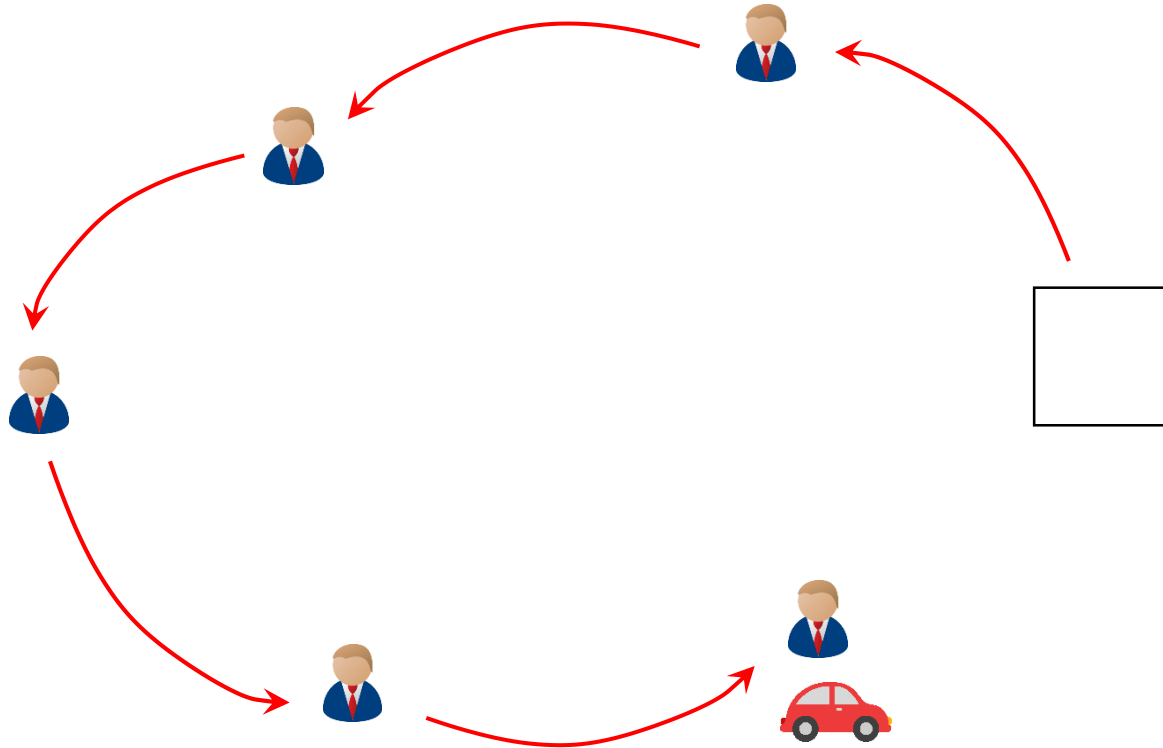
traveling salesman problem



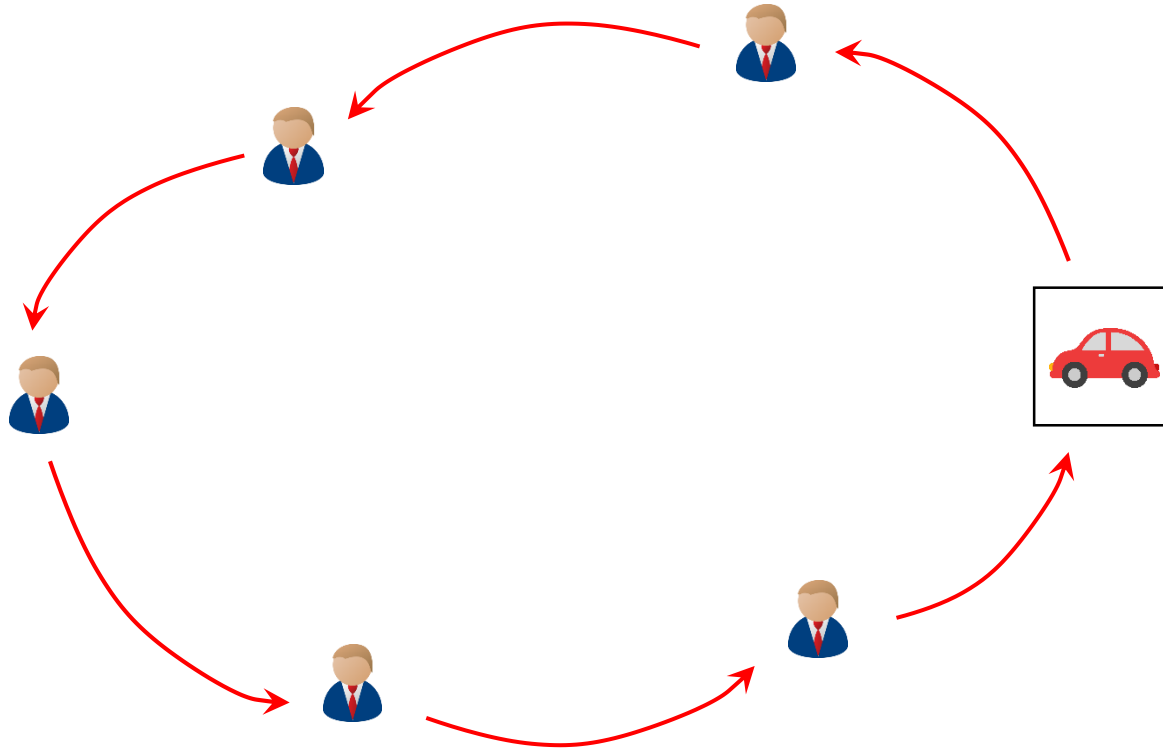
traveling salesman problem



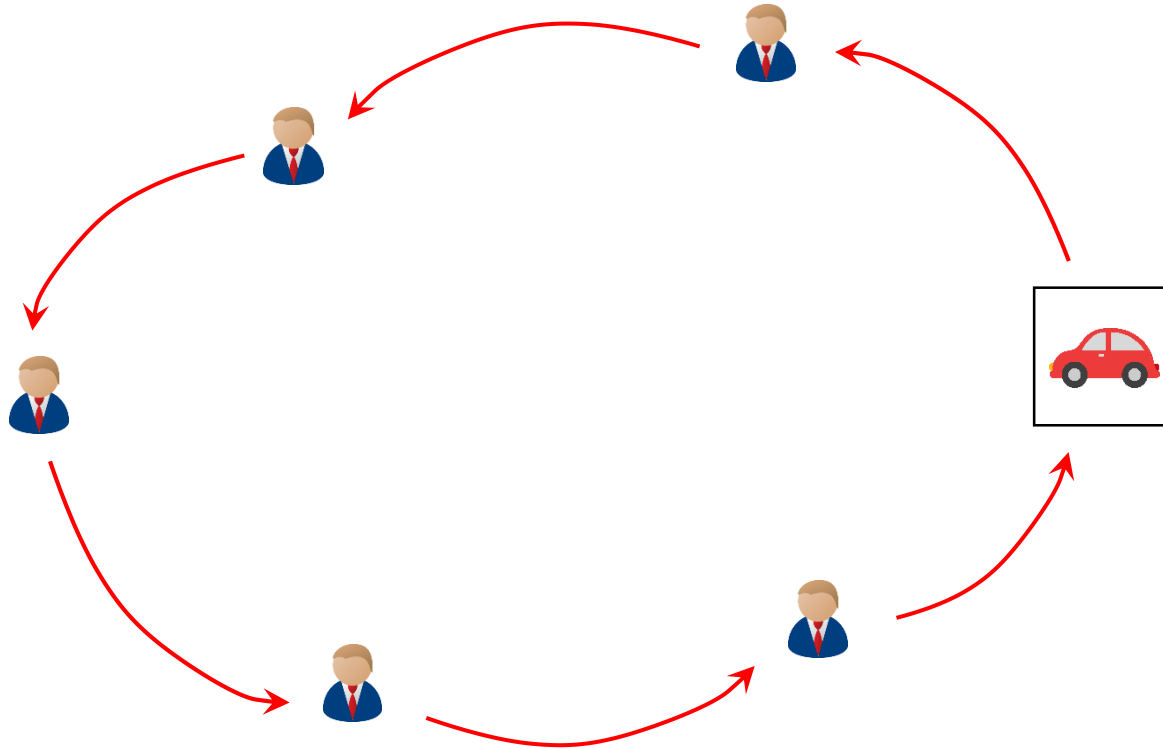
traveling salesman problem



traveling salesman problem

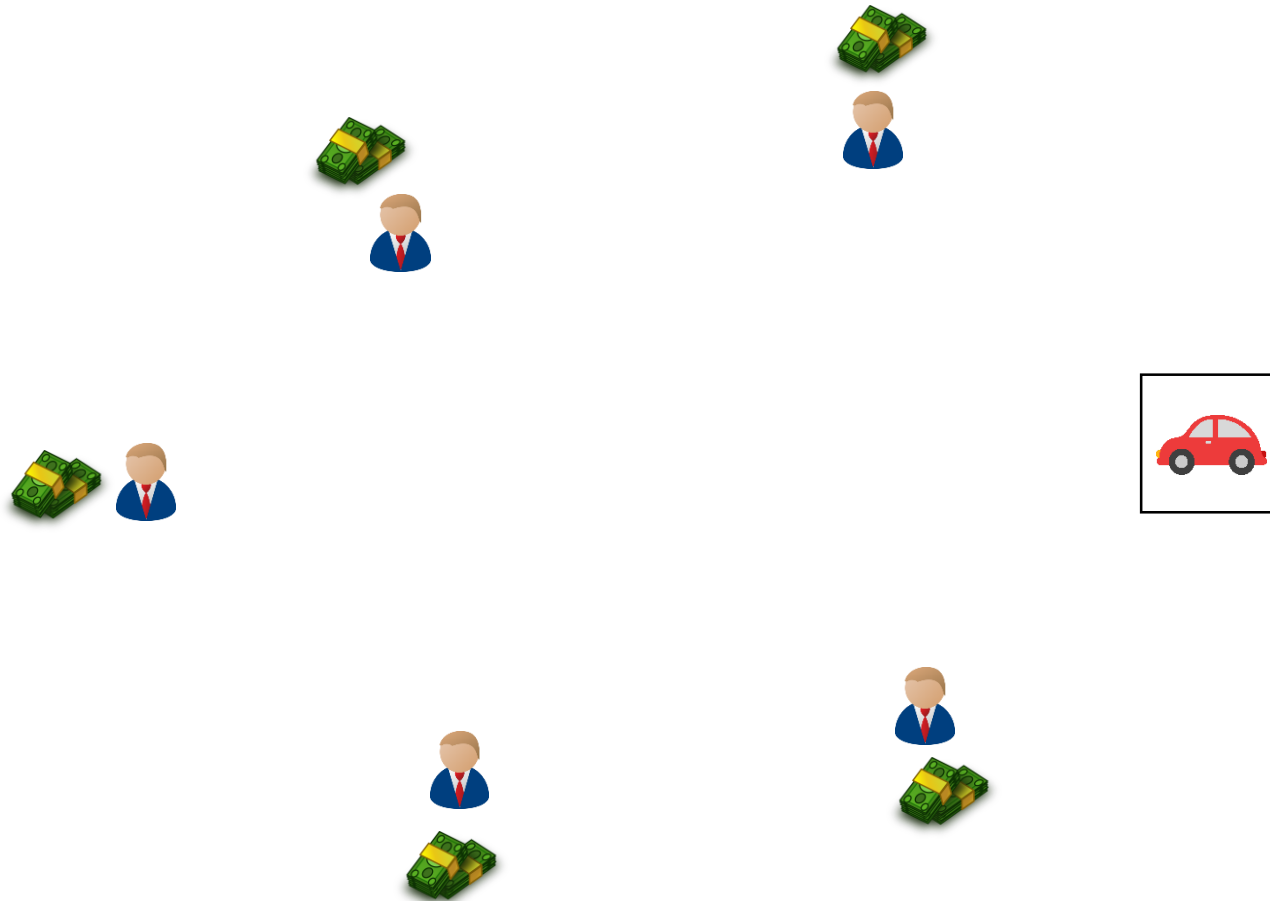


traveling salesman problem

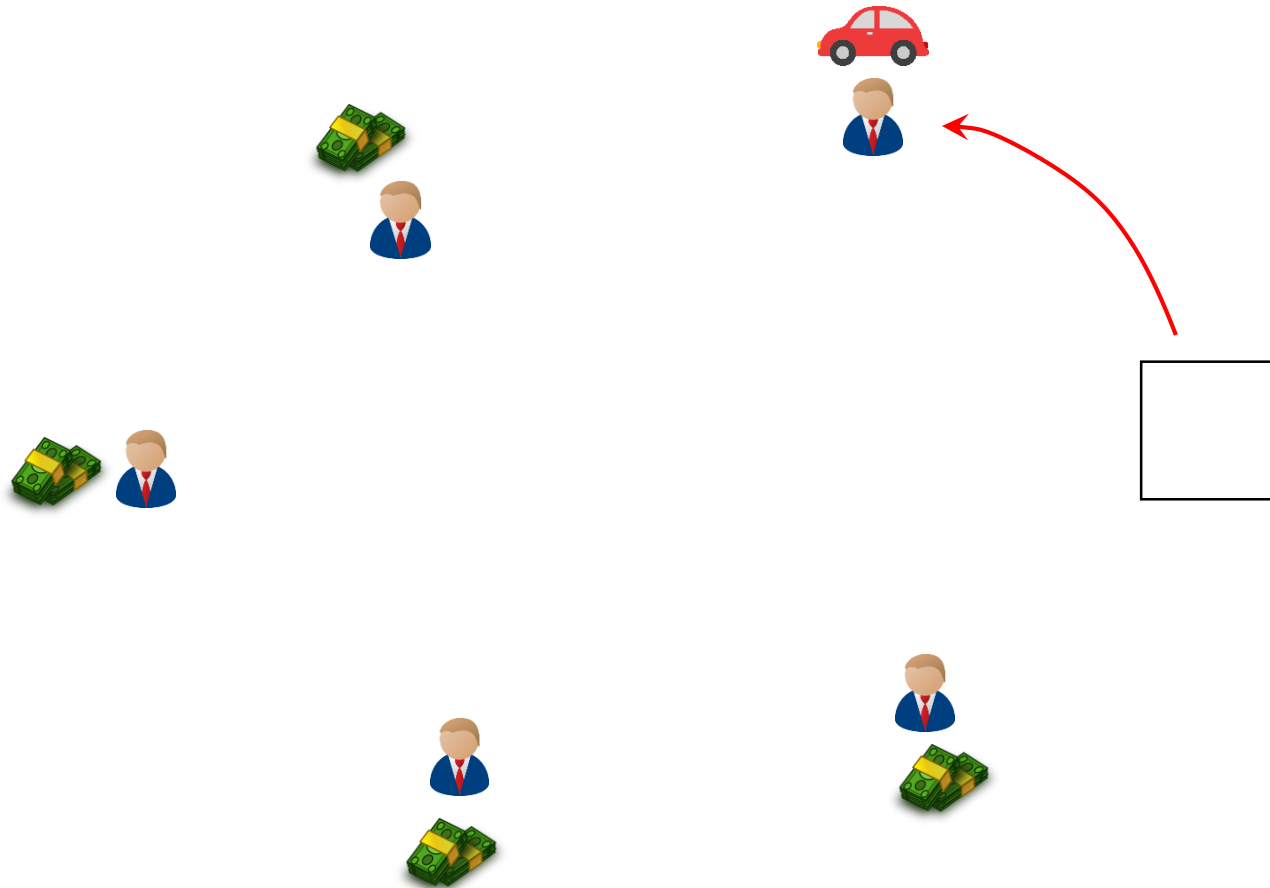


minimizing cost

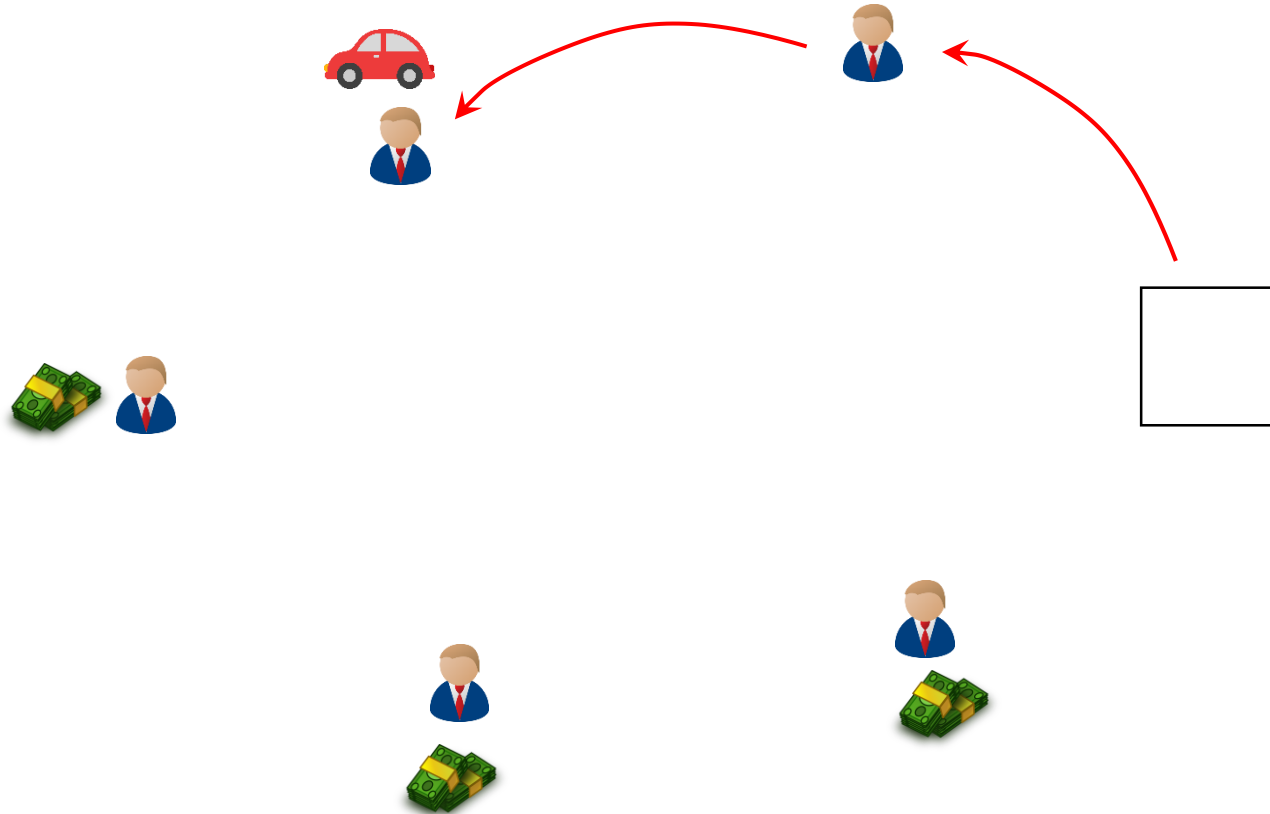
traveling salesman problem with profits (profitable tour problem)



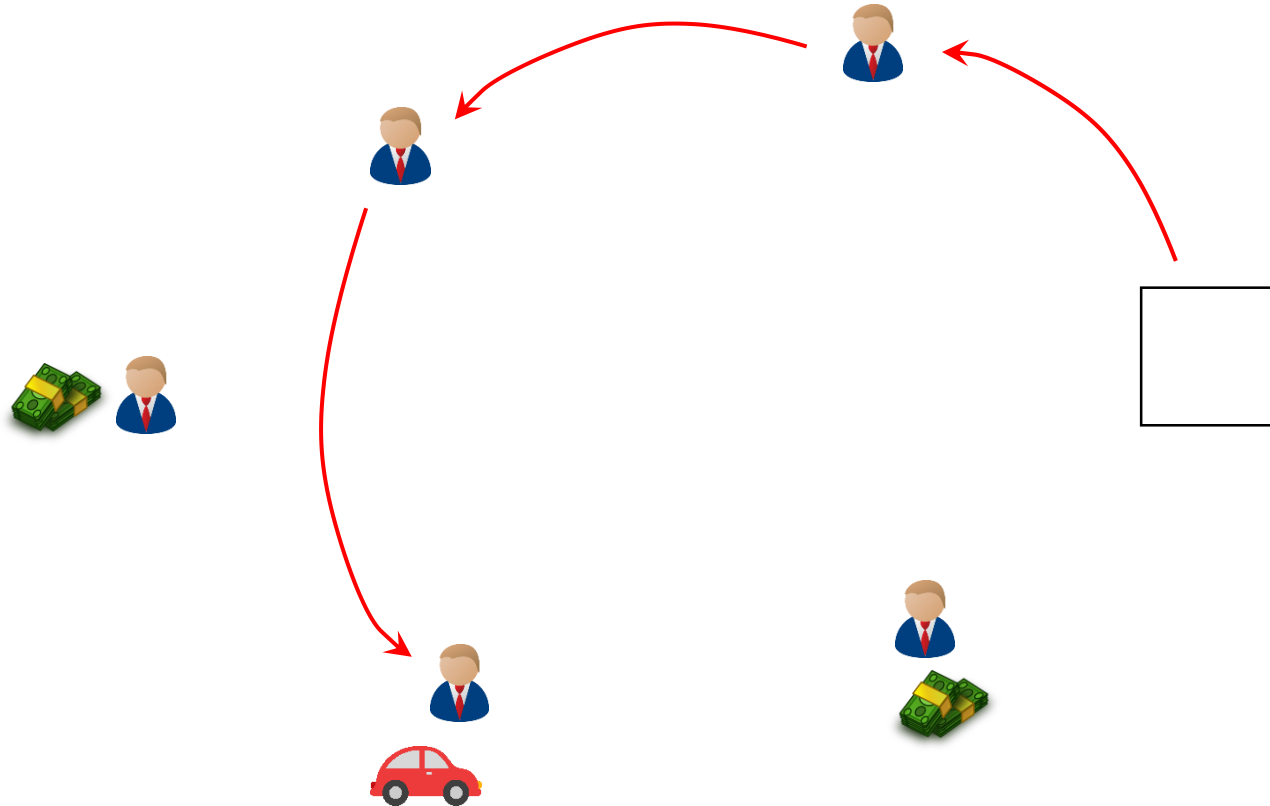
traveling salesman problem with profits (profitable tour problem)



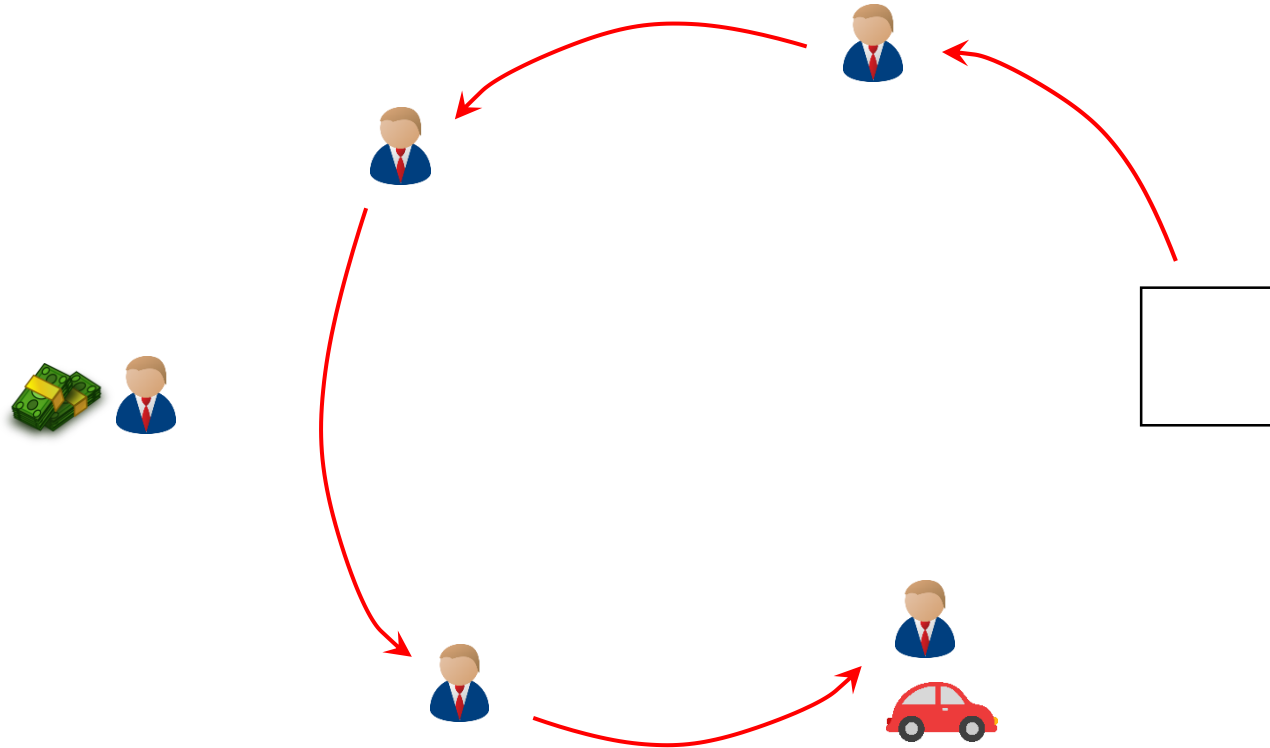
traveling salesman problem with profits (profitable tour problem)



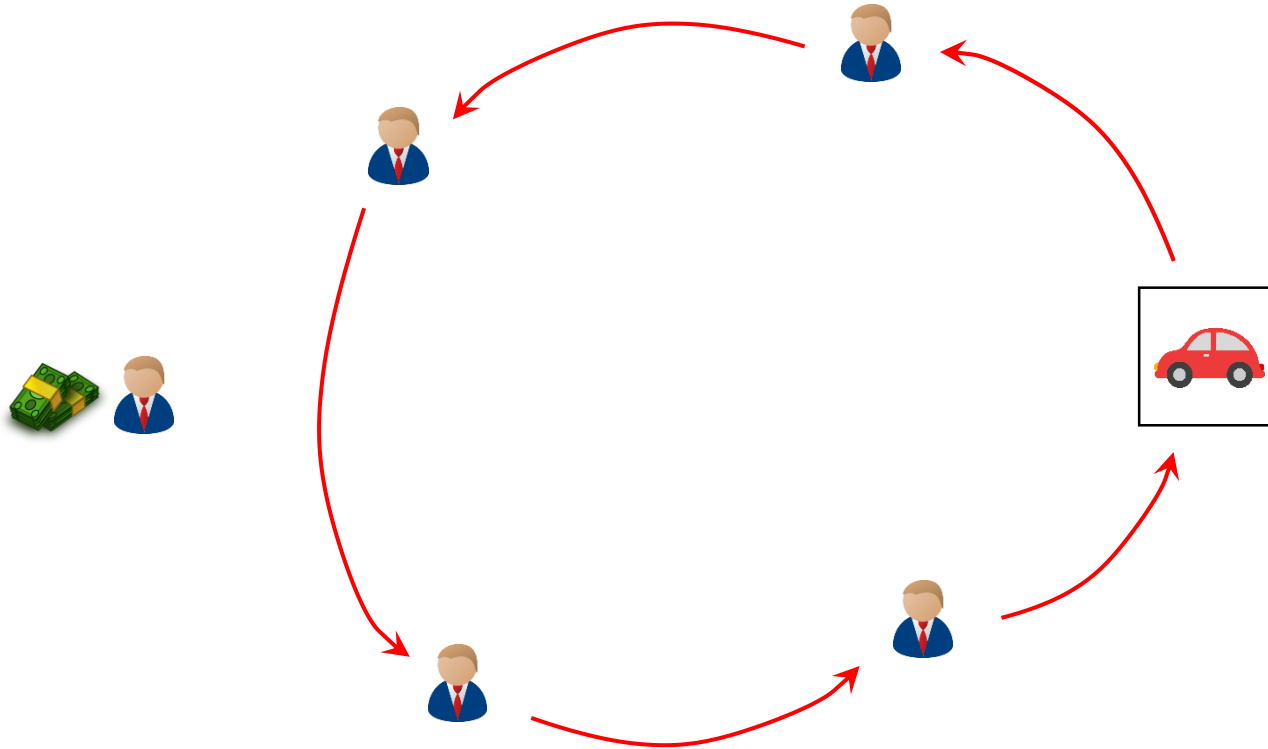
traveling salesman problem with profits (profitable tour problem)



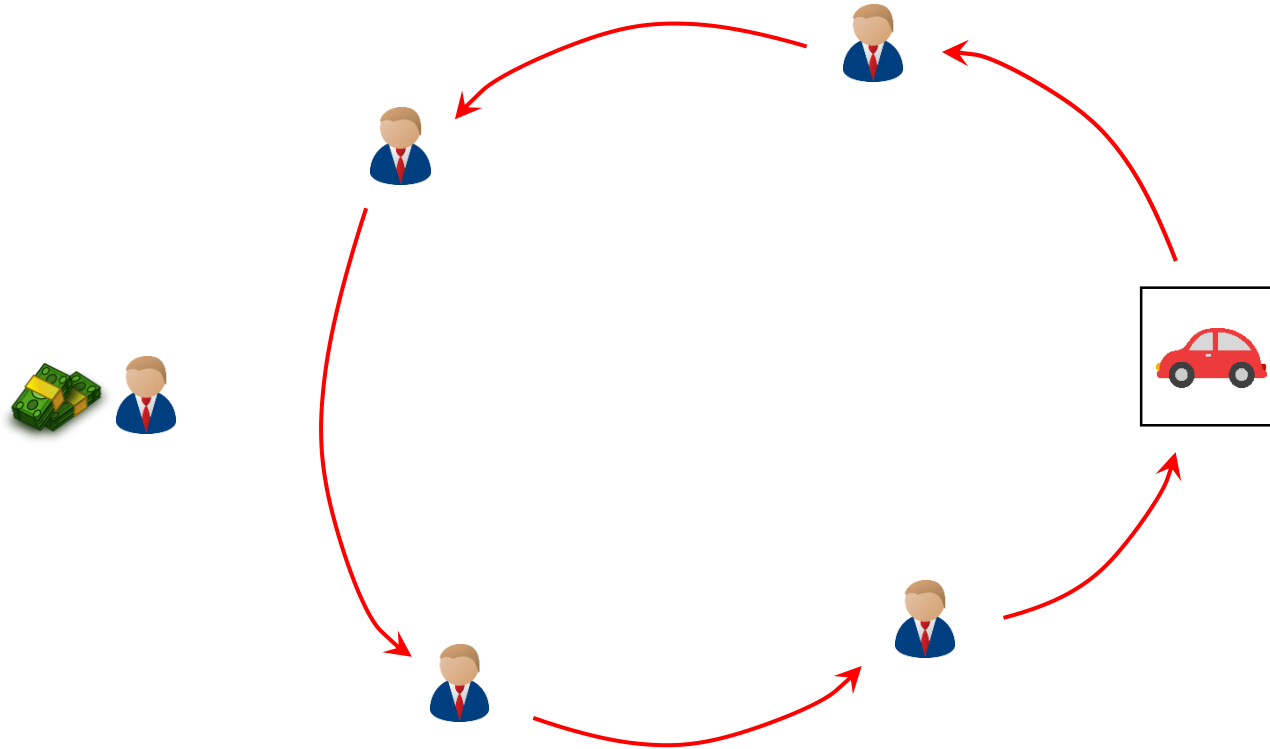
traveling salesman problem with profits (profitable tour problem)



traveling salesman problem with profits (profitable tour problem)



traveling salesman problem with profits (profitable tour problem)



maximizing (profit - cost)

Cooperation of customers in traveling salesman problems with profits

Research question: What prizes should customers offer to ensure being visited?

Methodology: Cooperative game theory.

Outcome: Exploring the cooperative potential in traveling salesman problems and providing a tool to evaluate individual customers as well as groups of customers.

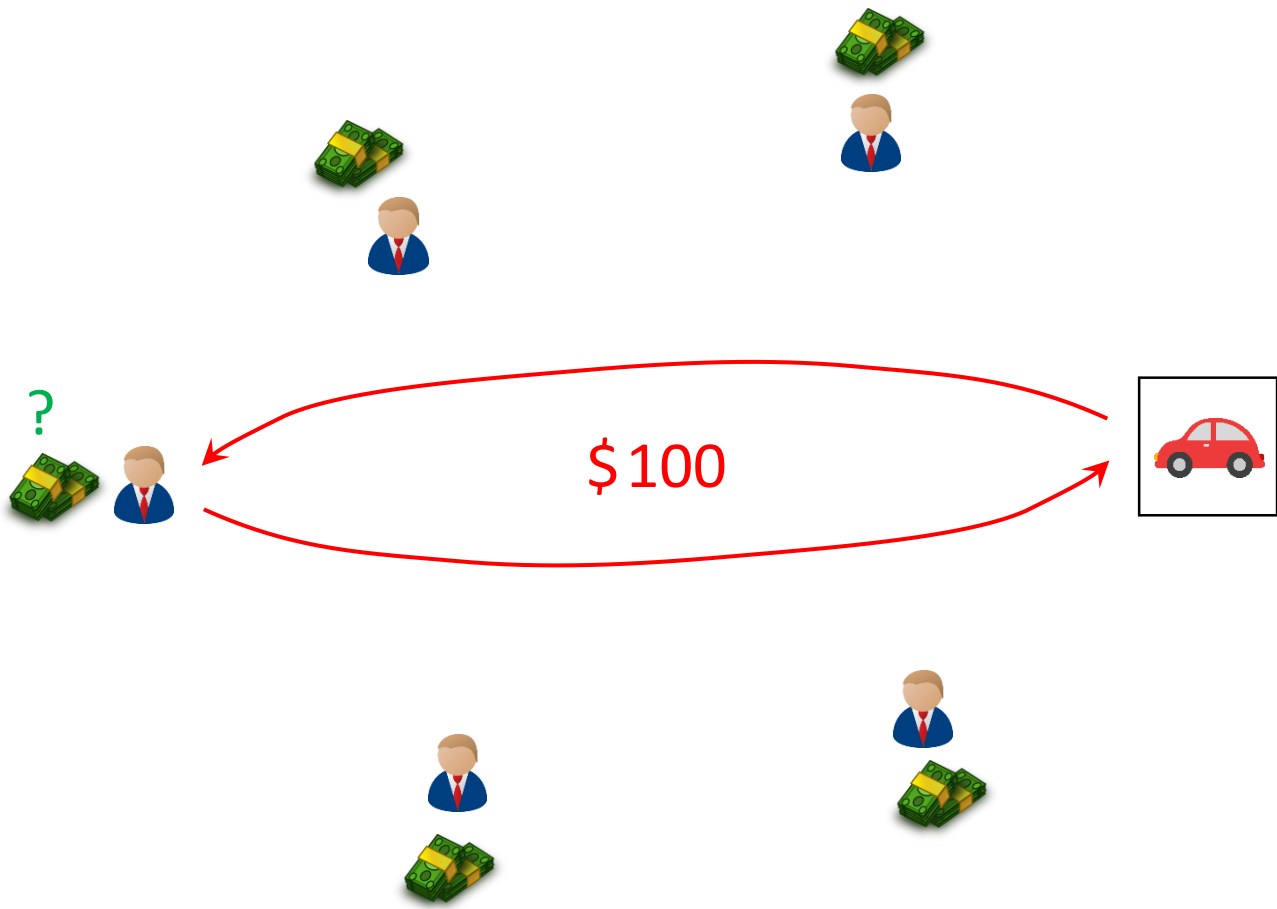
Journal: Currently under review in *Optimization Letters*.



What prizes should the customers offer to ensure being visited?



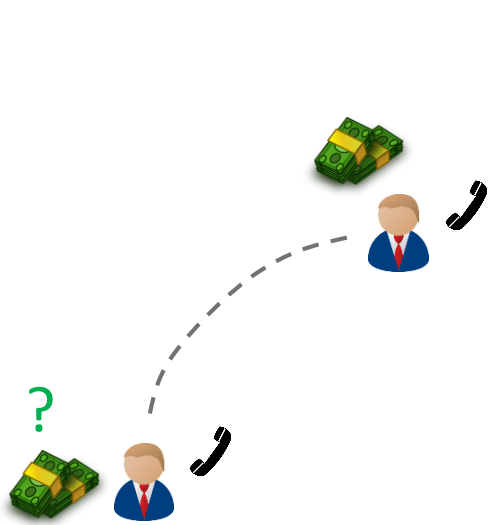


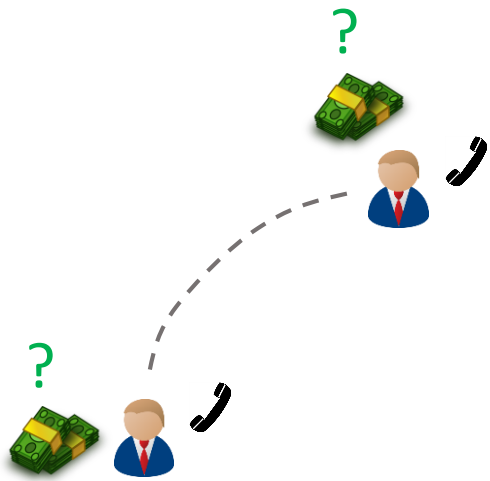


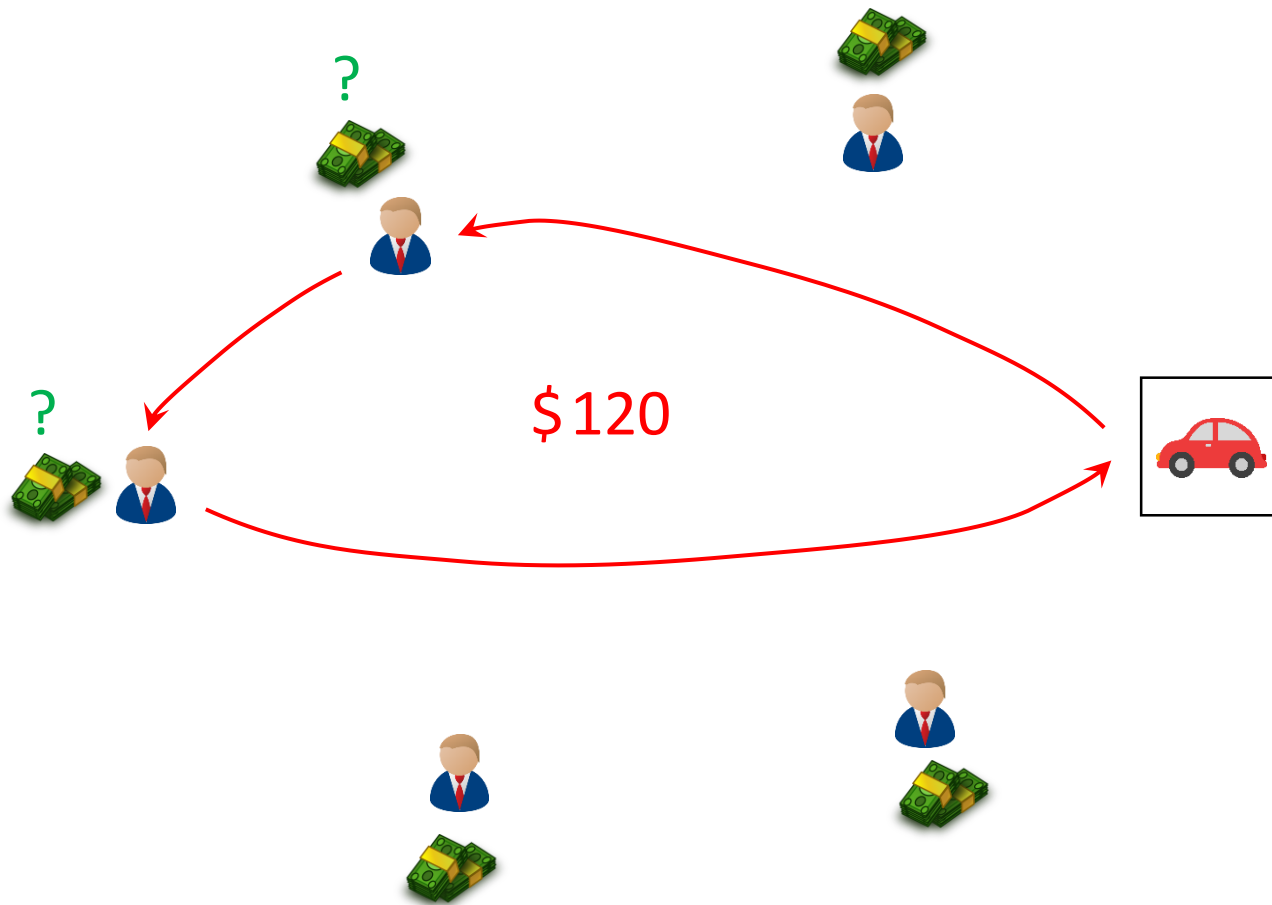
$\geq \$100$

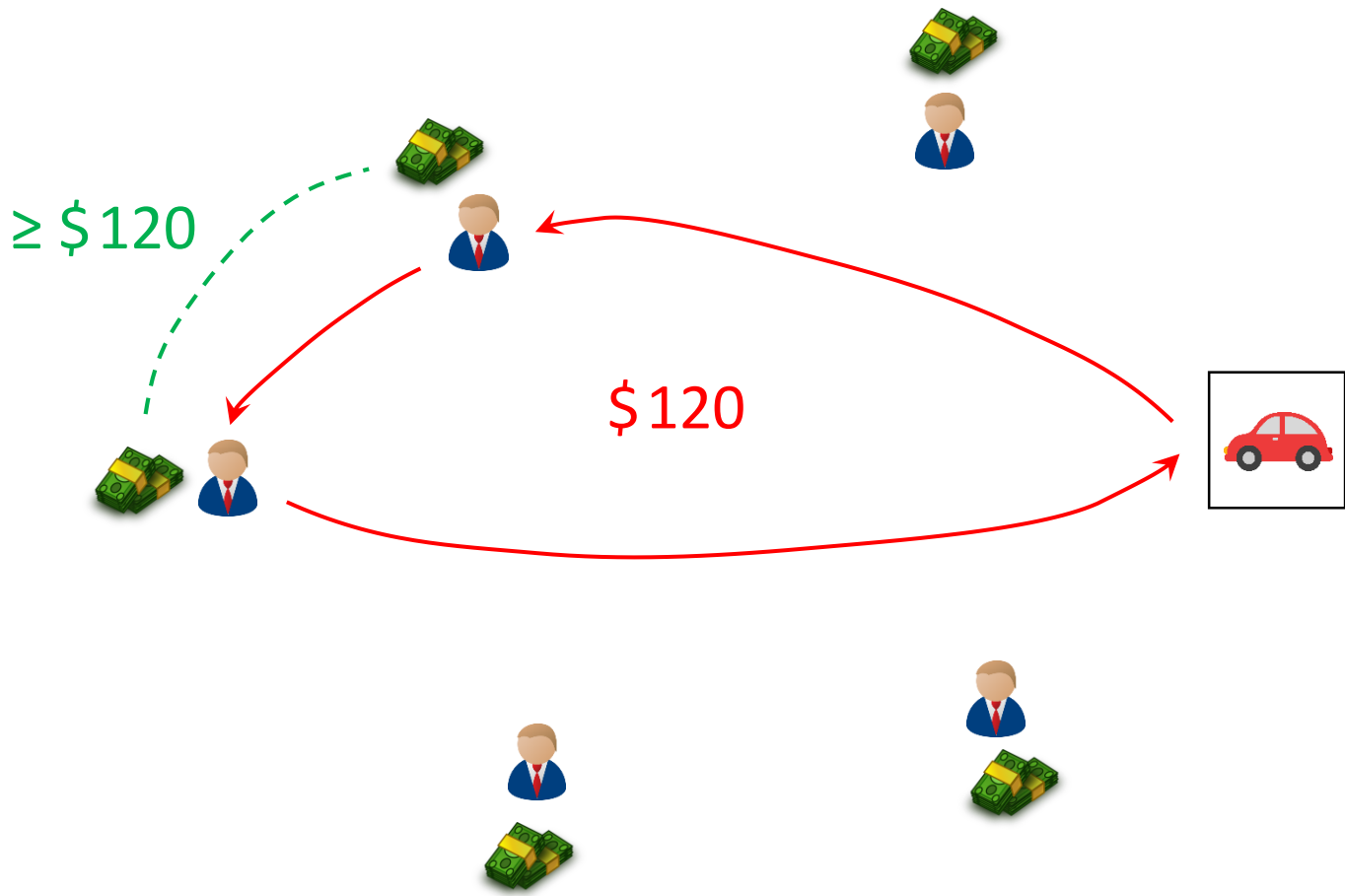




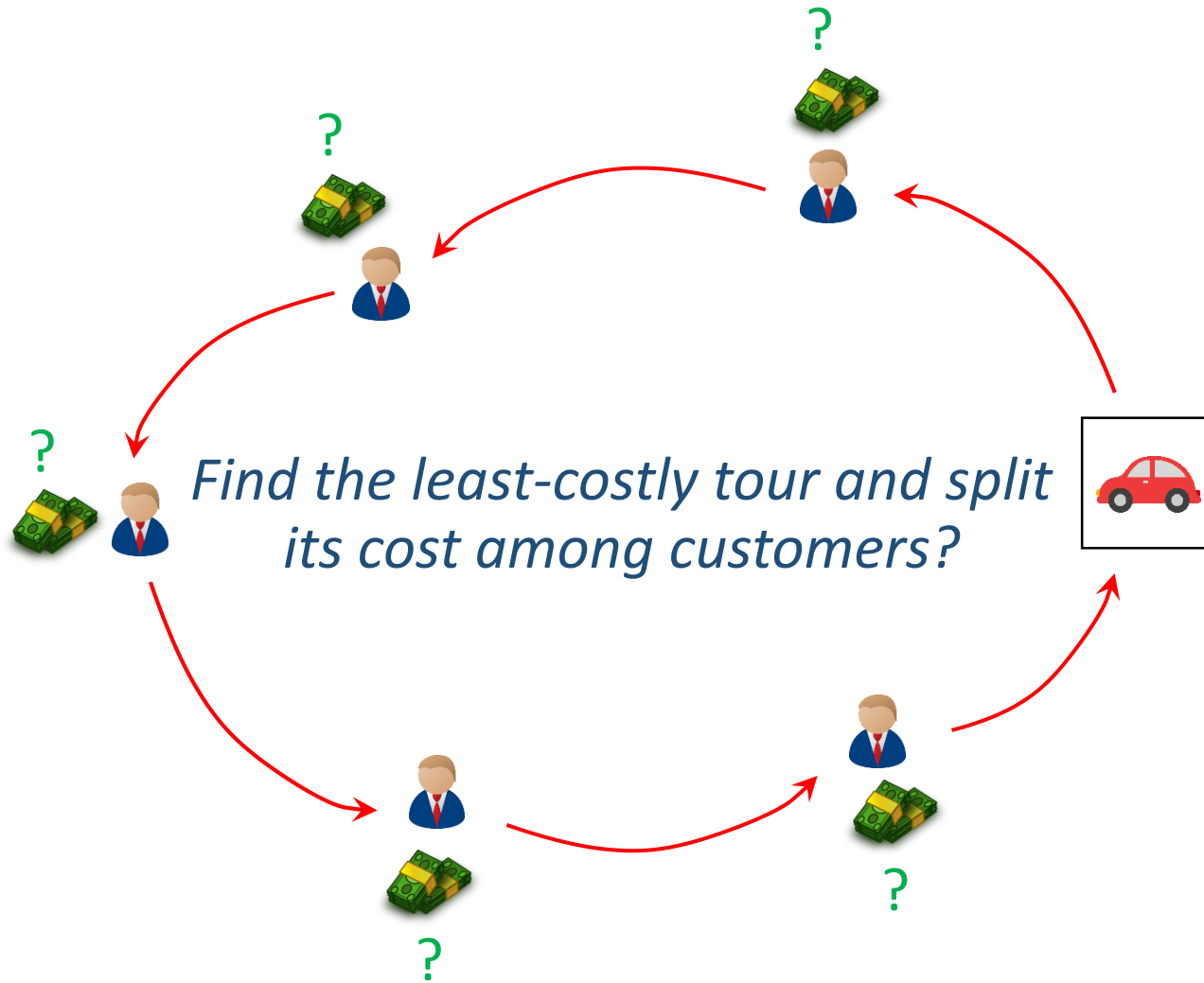












If this was true, then

traveling salesman games

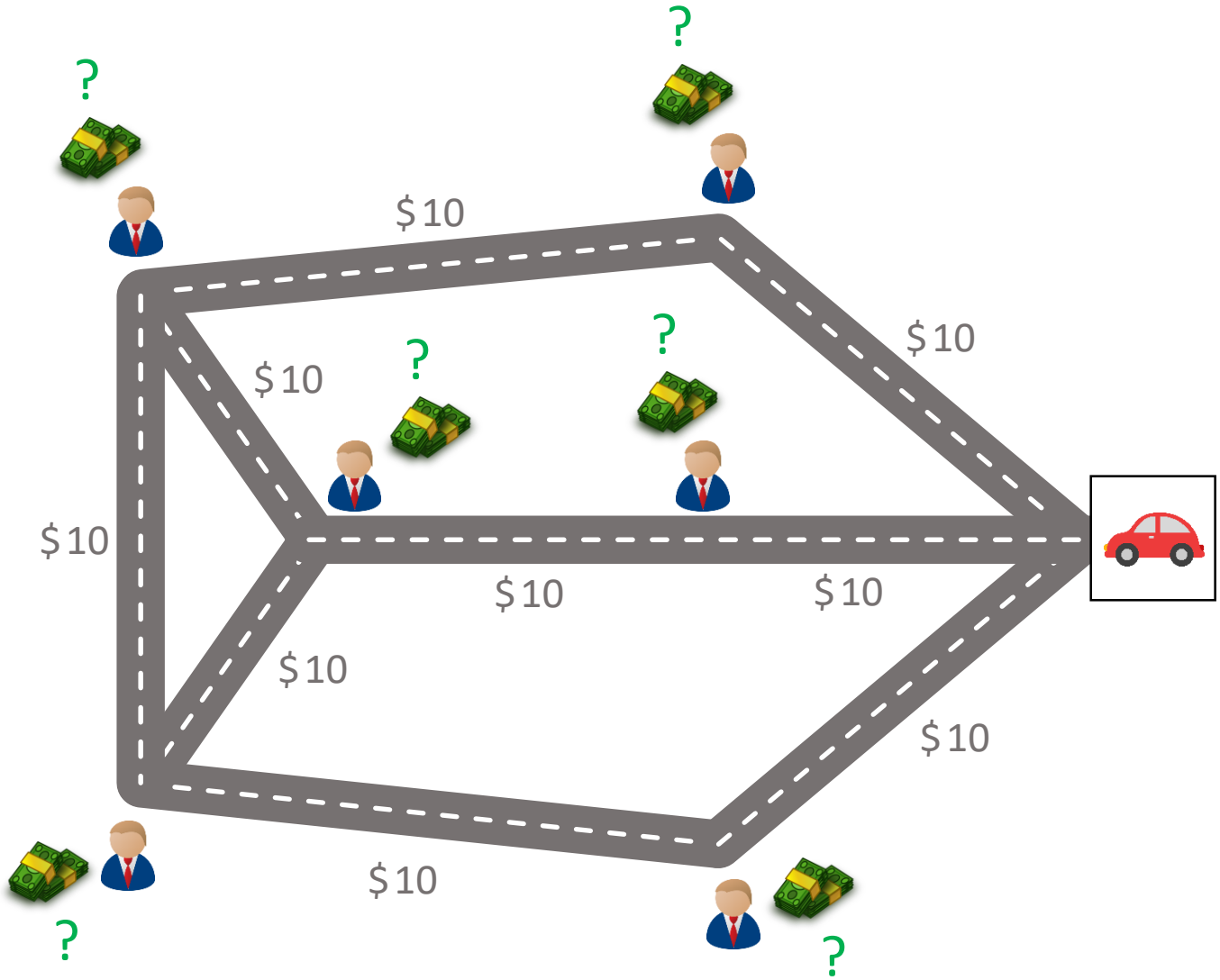
(Potters et al., 1992)

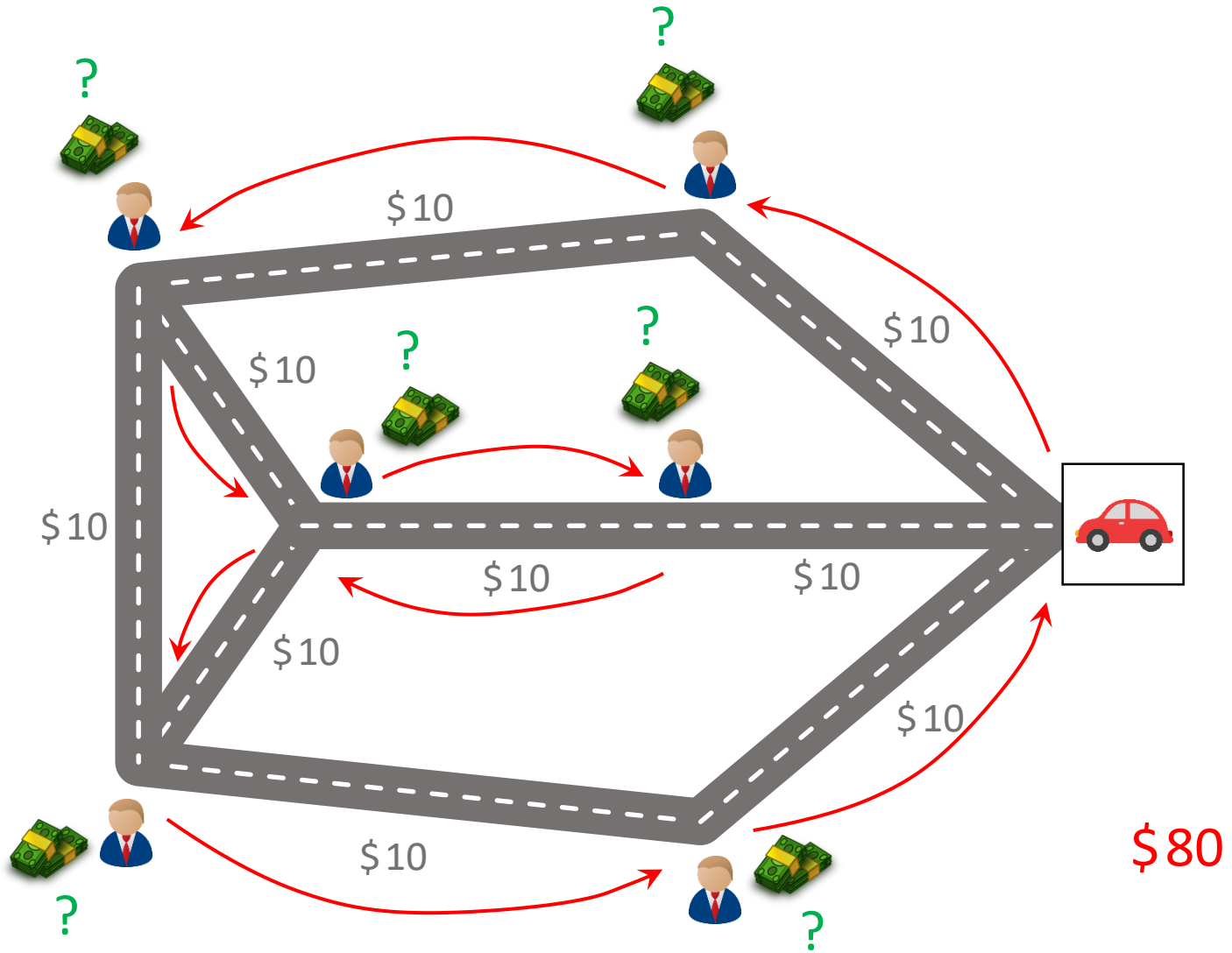
or

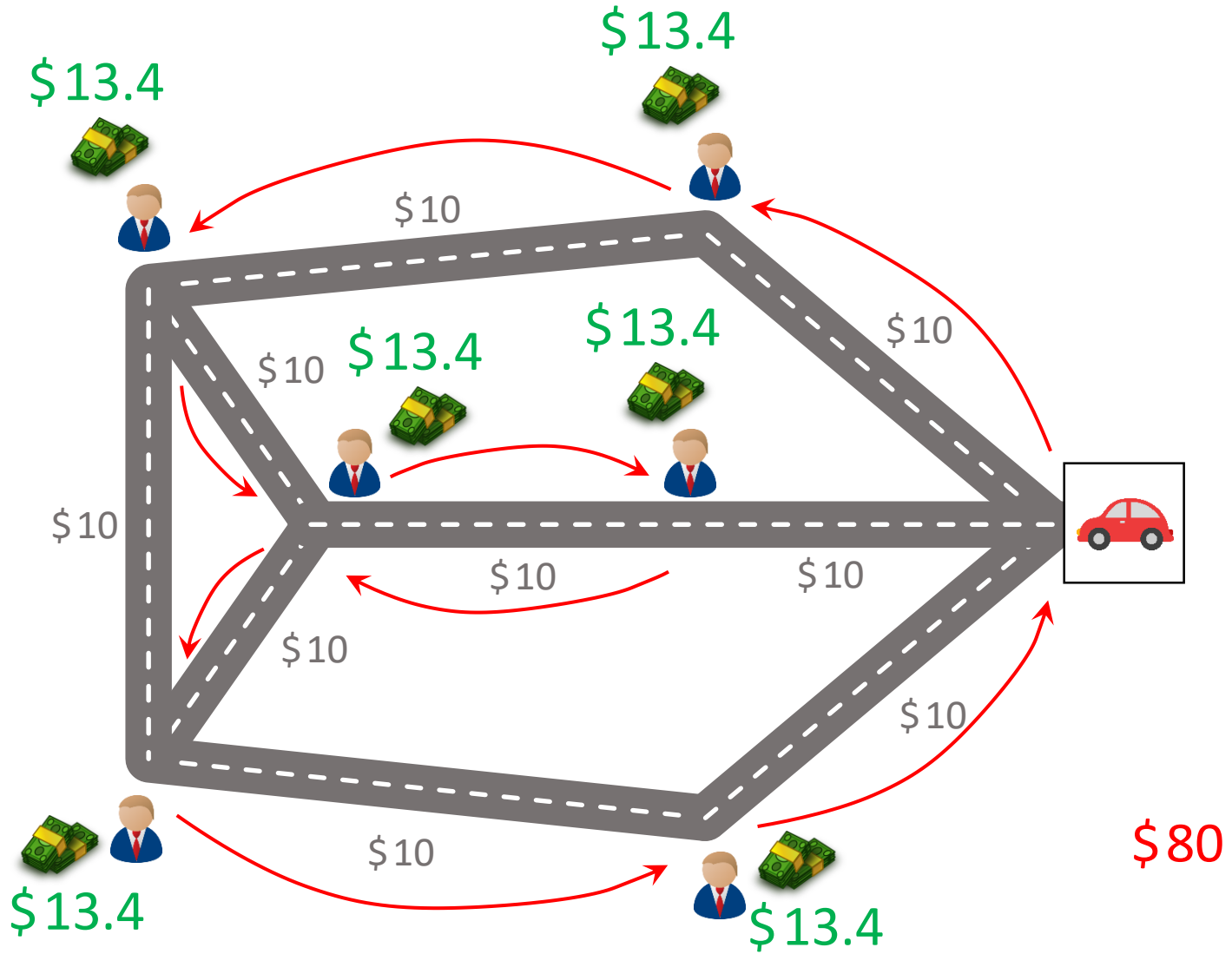
routing games with revenues

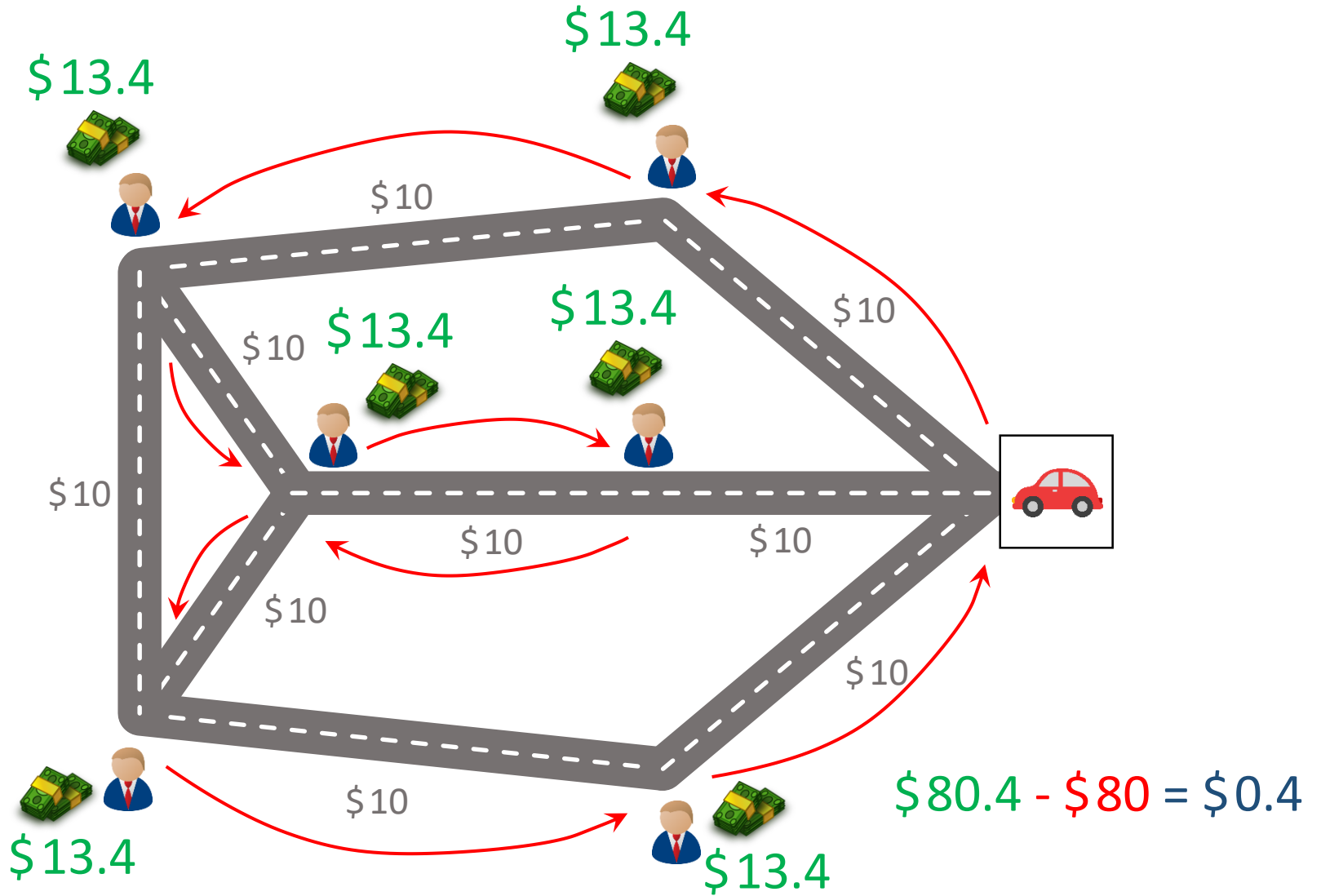
(Estévez-Fernández et al., 2009)

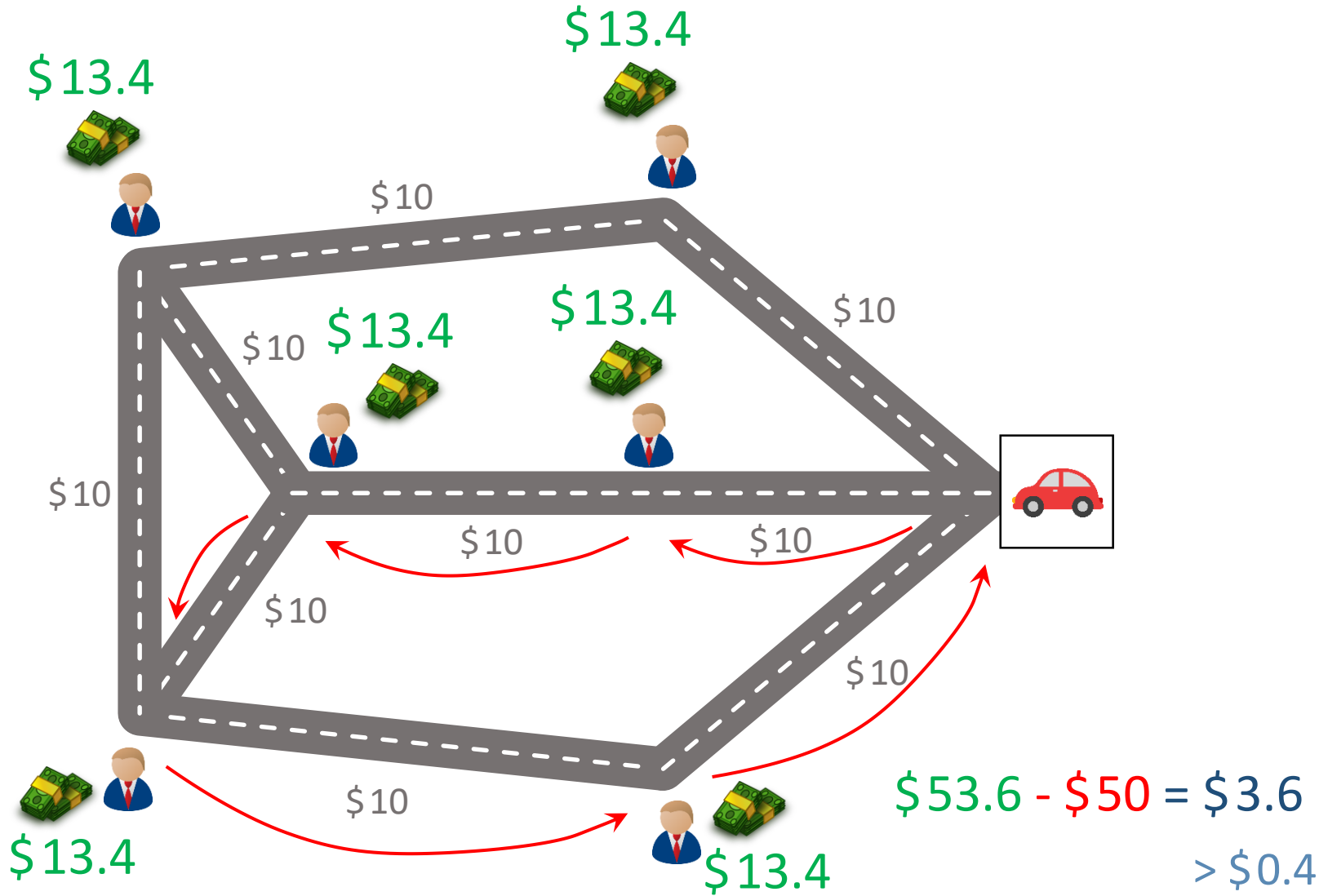
would be sufficient to describe the problem.

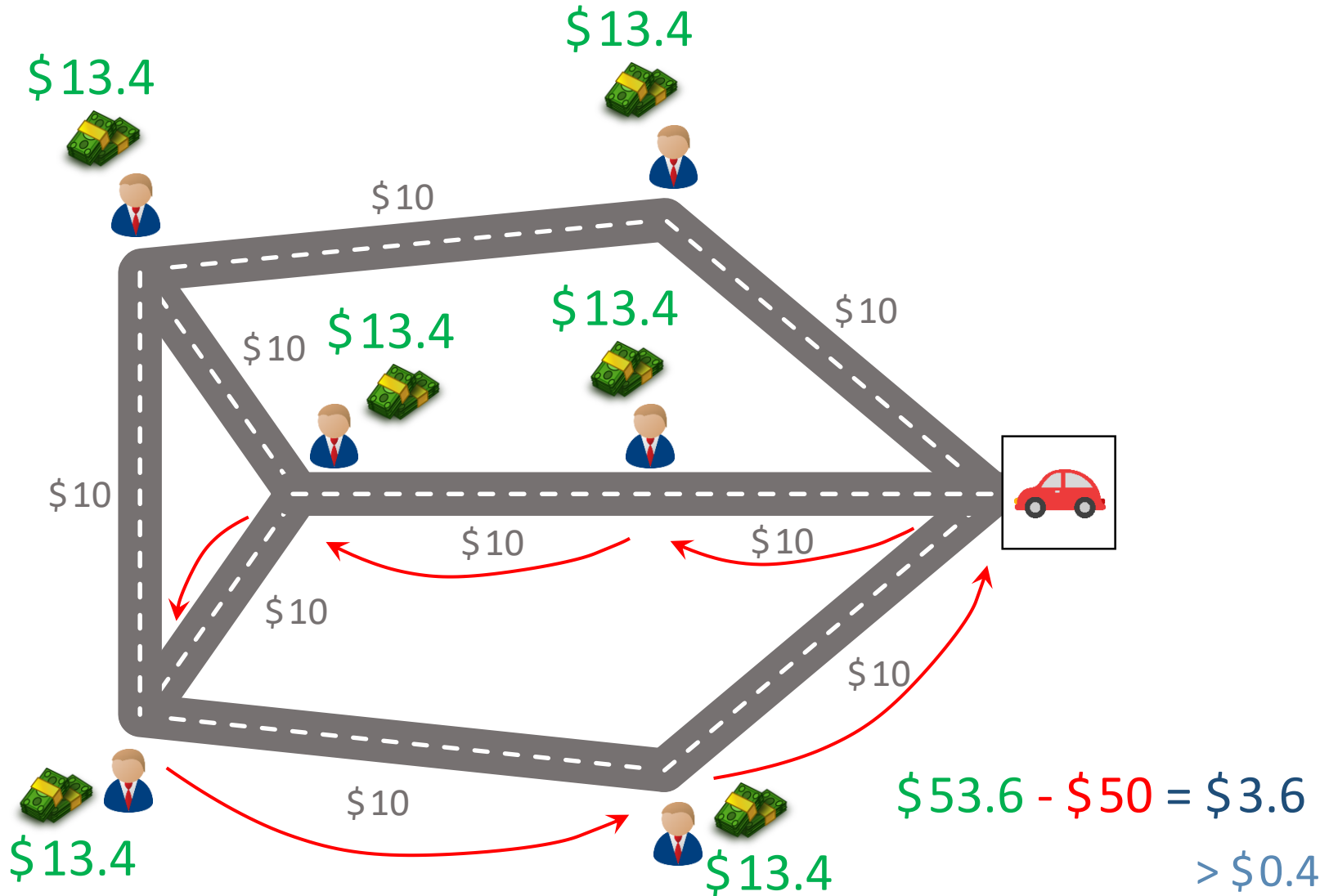












no way to divide \$80 such that all customers would be visited

to determine how much to allocate

to determine how much to allocate

profitable tour game (N, Cost^{PTP})

$$\begin{aligned} \text{Cost}^{PTP}(S) &= \min_{\text{Prize}_j} \sum_{j \in S} \text{Prize}_j \\ \text{s.t. } \sum_{j \in S \setminus R} \text{Prize}_j &\geq \text{Cost}^{TSP}(S \cup T) - \text{Cost}^{TSP}(R \cup T) && \forall R \subseteq S, \forall T \subseteq N \setminus S, \\ \text{Prize}_j &\geq 0 && \forall j \in S. \end{aligned}$$

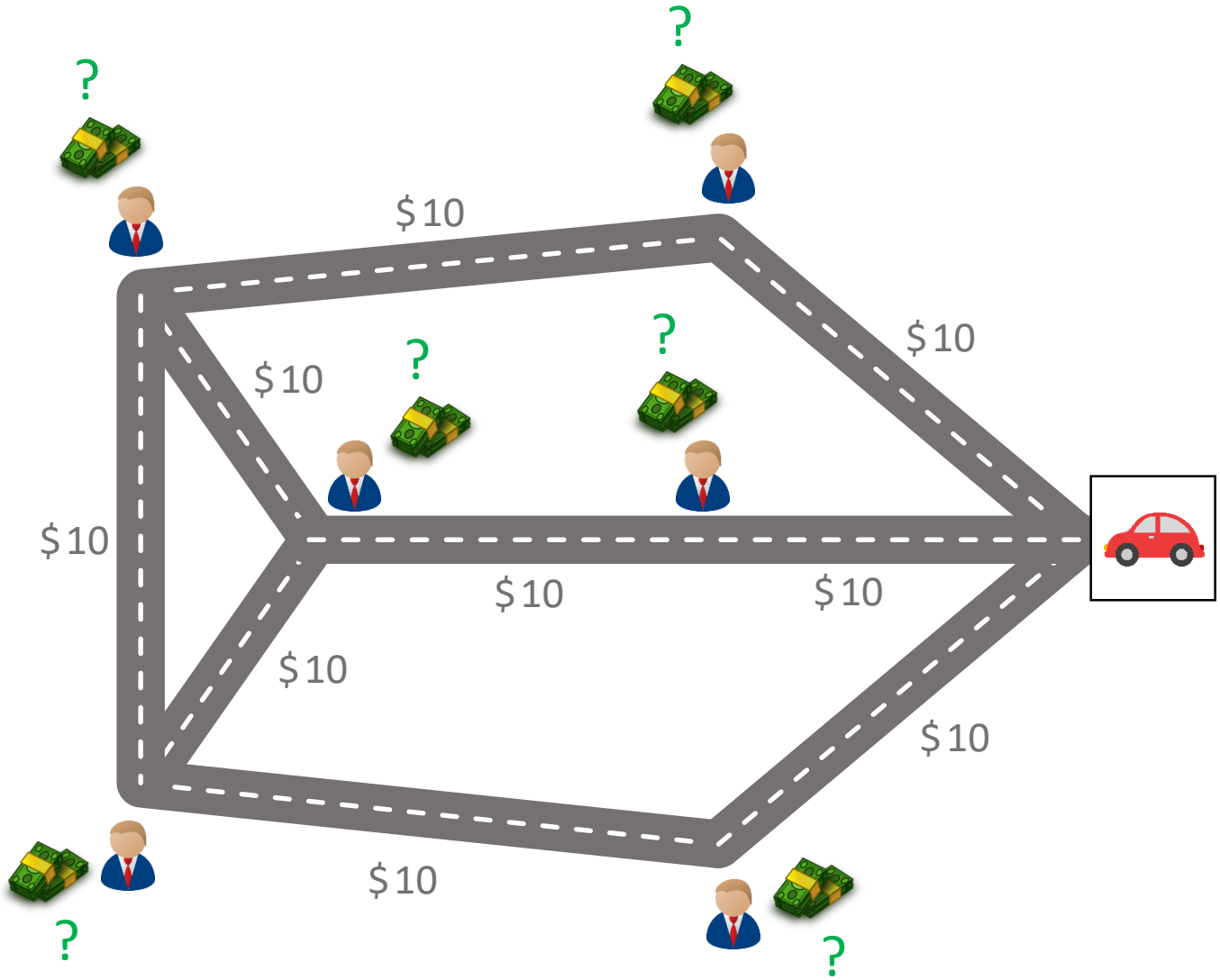
to determine how to allocate it

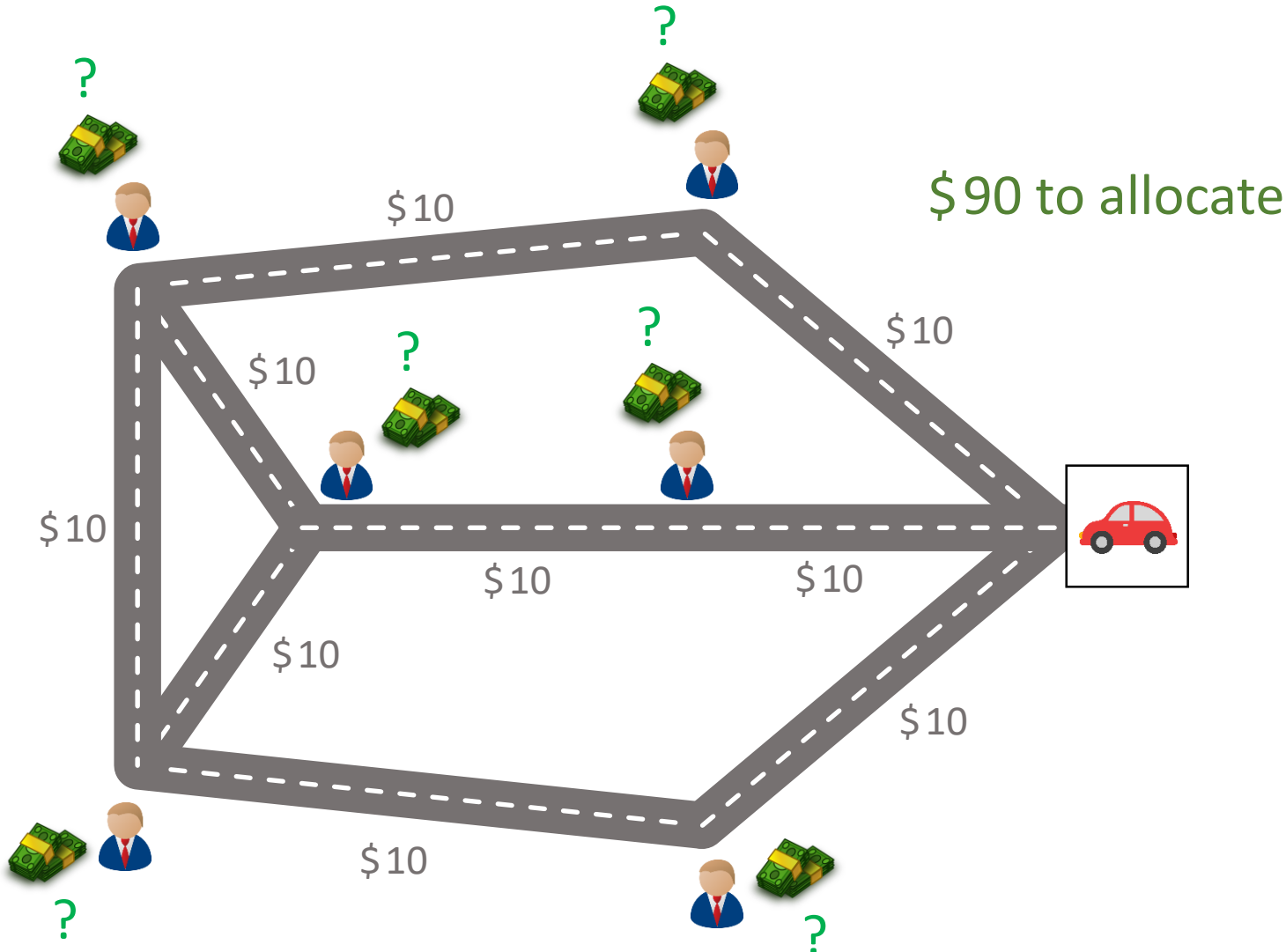
to determine how to allocate it

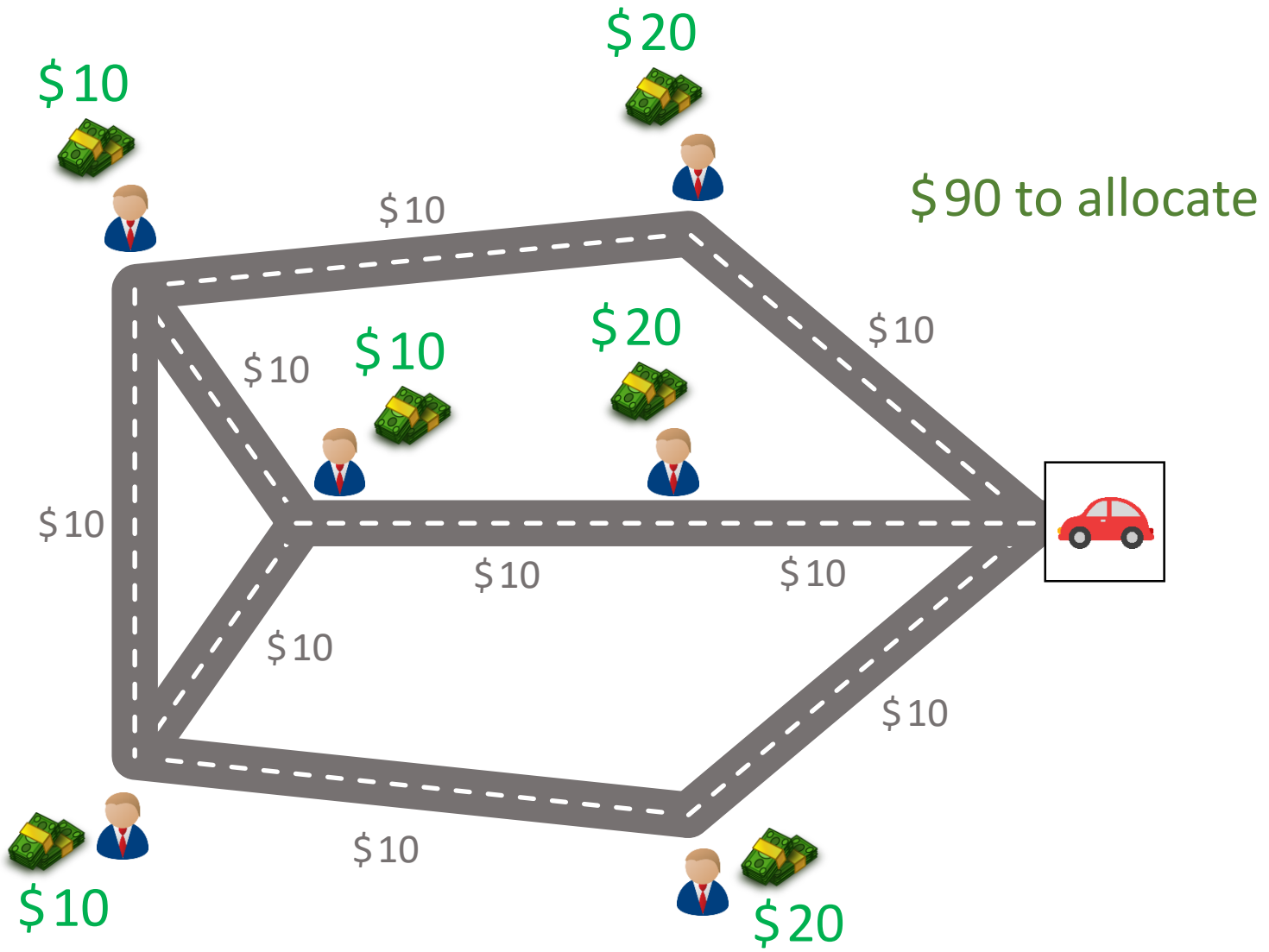
Theorem 1 *If the core of the traveling salesman game (N, Cost^{TSP}) is nonempty, all allocations from the core, and no other, are optimal prize allocations of the grand coalition in the profitable tour game (N, Cost^{PTP}) .*

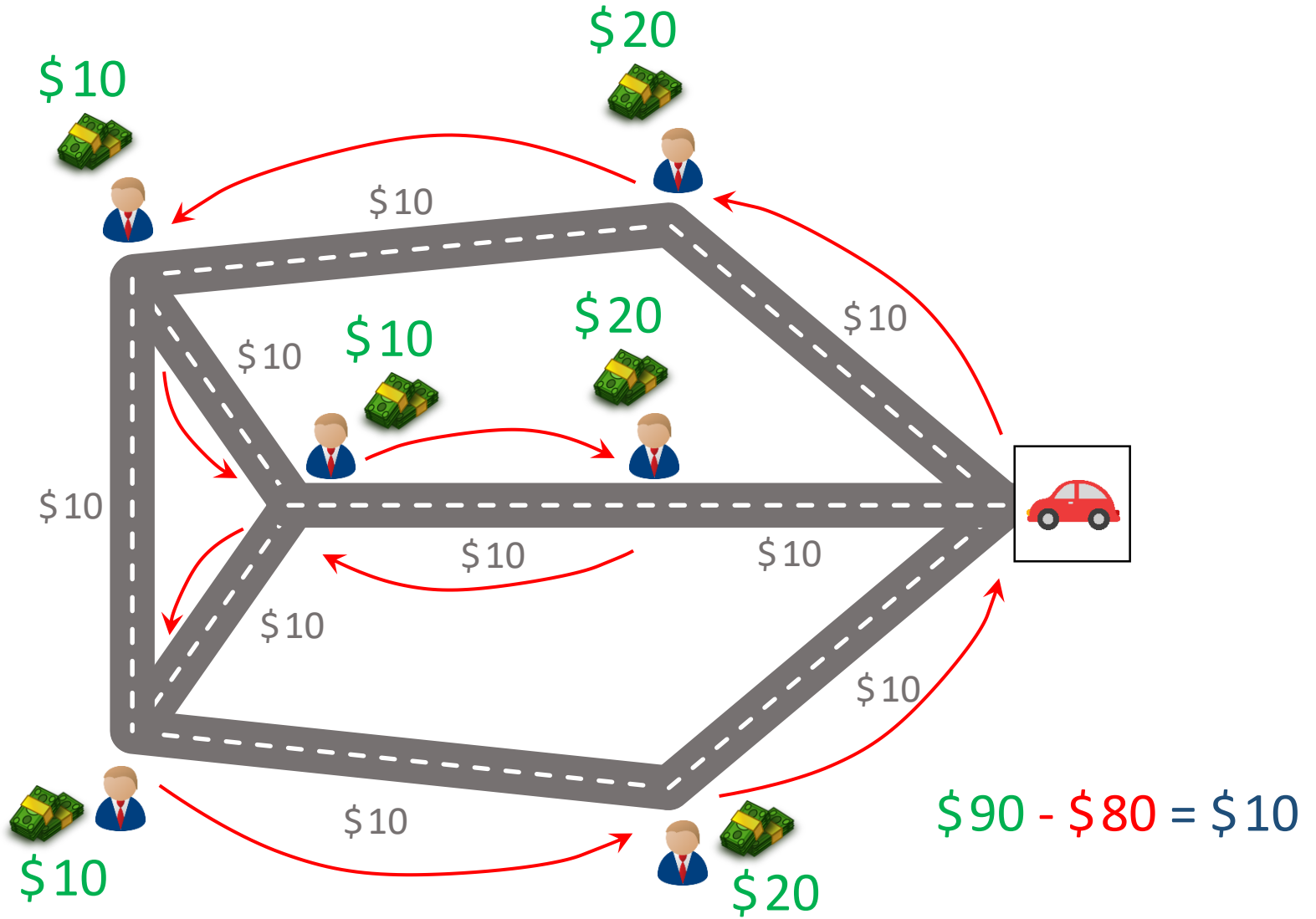
Theorem 2 *If the core of the traveling salesman game (N, Cost^{TSP}) is empty, then prizes Prize_j stand for an optimal prize allocation of the grand coalition in the profitable tour game (N, Cost^{PTP}) if and only if they represent an optimal solution of problem*

$$\begin{aligned} & \min_{\text{Prize}_j, \varepsilon} \varepsilon \\ \text{s.t. } & \sum_{j \in N} \text{Prize}_j = \text{Cost}^{TSP}(N) + \varepsilon, \\ & \sum_{j \in S} \text{Prize}_j \leq \text{Cost}^{TSP}(S) + \varepsilon \quad \forall S \subset N, \\ & \text{Prize}_j \geq 0 \quad \forall j \in N, \\ & \varepsilon \geq 0. \end{aligned}$$

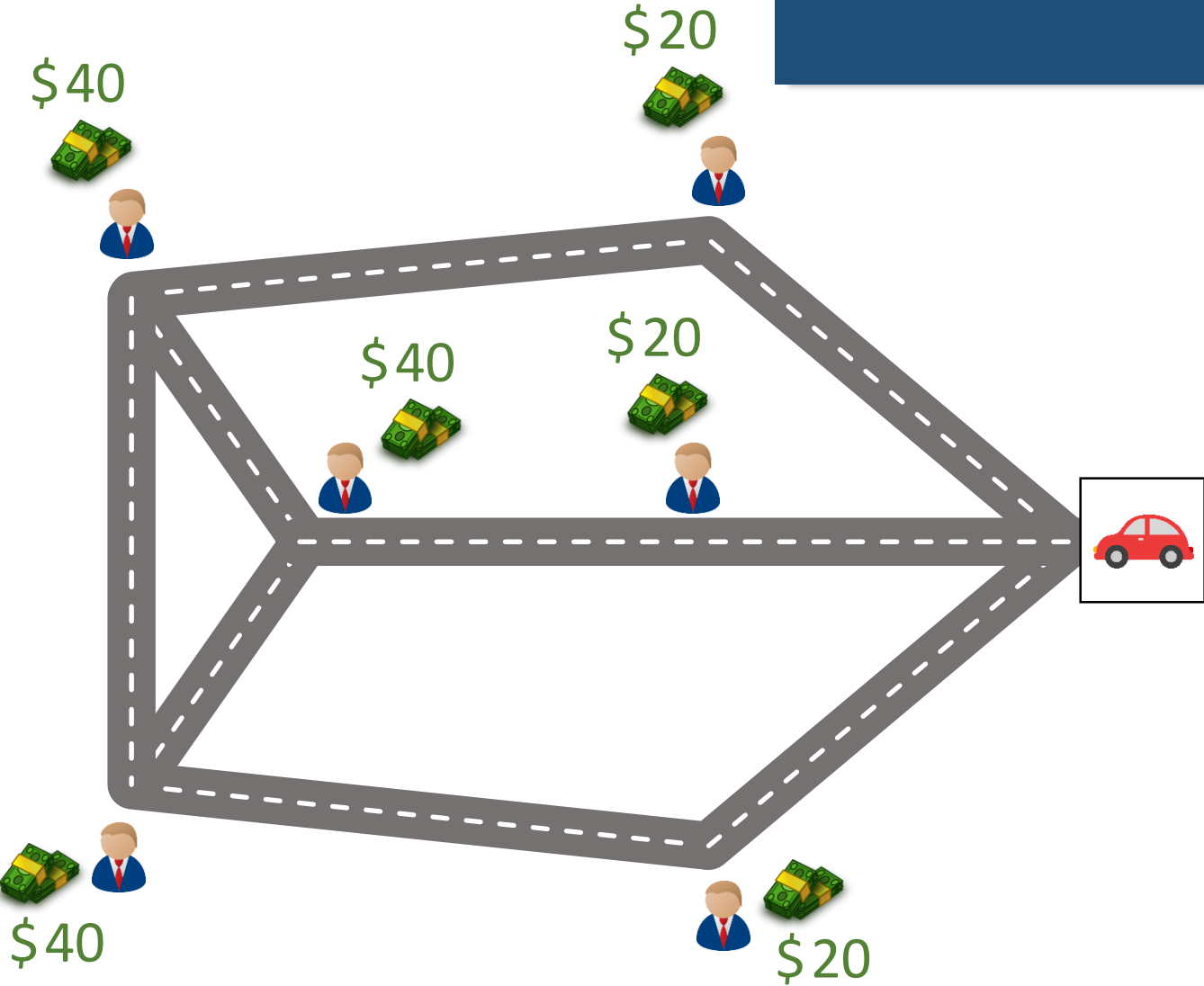




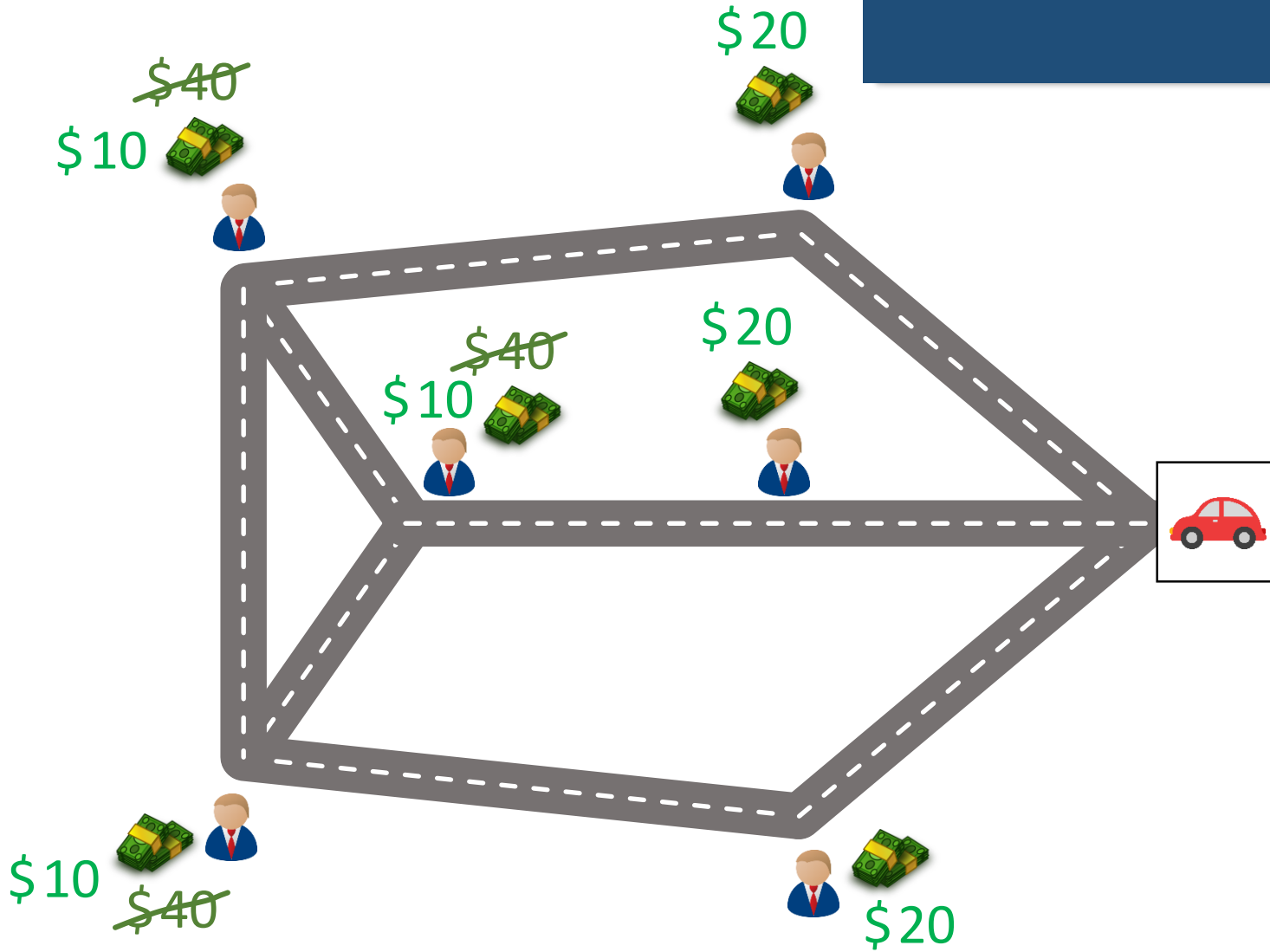




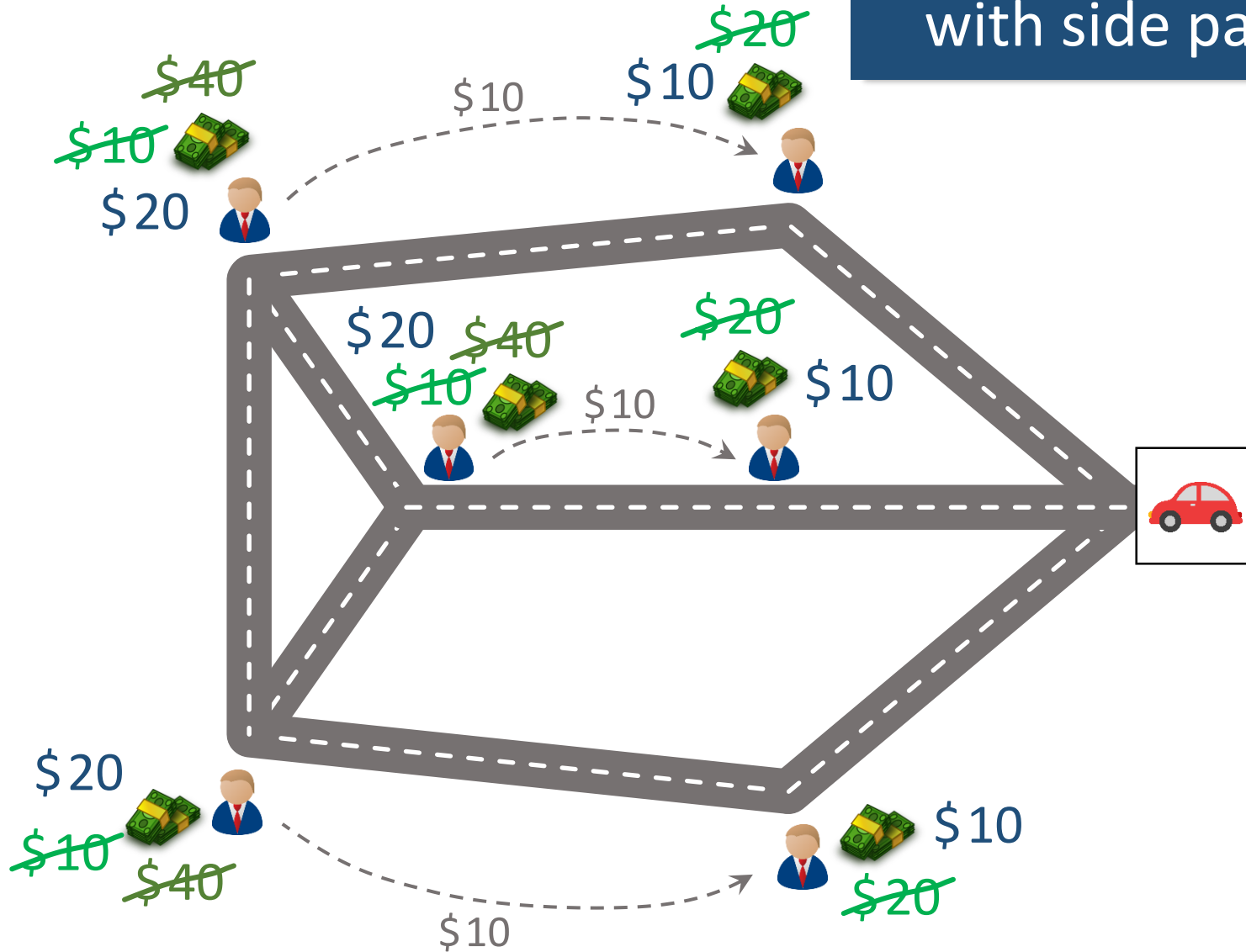
noncooperative case



cooperative case



cooperative case with side payments

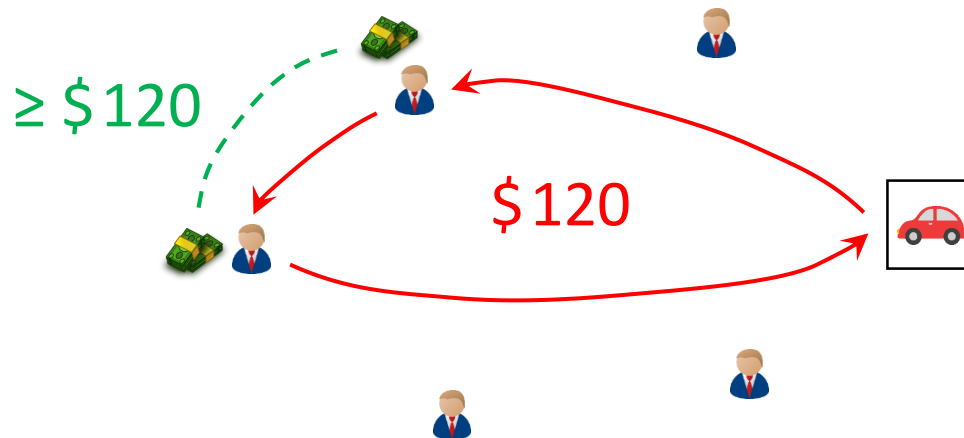


→ *cost allocation in profitable tour games*

Applications

savings in cooperation of customers

Customers make orders together to achieve lower transportation costs.



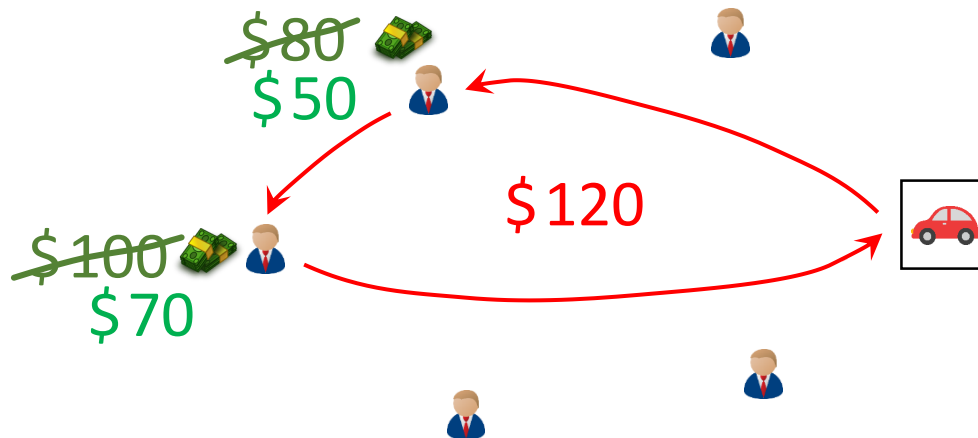
Prerequisites: Customers need to reveal their intentions to make an order to other customers.

Customers should not make the orders before all intentions are revealed.

The carrier must be able to satisfy several orders within a single route.

discounts on transportation costs

Carrier offers tailored transportation cost discounts on more order from the same area.



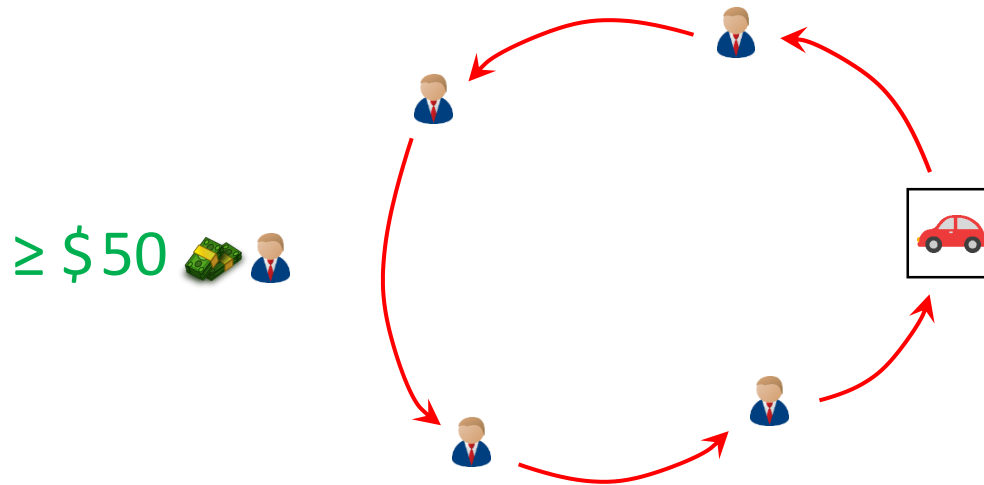
Prerequisites: Customers need to make orders before knowing the actual cost (knowing only the worst-case cost).

Carrier willing to offer discount after the orders are made (appreciating current customers, not attracting new).

The carrier must be able to satisfy several orders within a single route.

evaluation and pricing of customers

Carrier can evaluate current customers as well as recognize additional cost of including new customers (groups of customers) to an existing transportation plan.



Prerequisite: New customers cannot cause a single-route plan to break into more routes.

Profitable Tour Game

cooperation of customers to achieve lower transportation costs

carrier offering tailored transportation cost discounts

evaluation and pricing of customers in an transportation plan



Thank you.

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